

# VIBRATION OF A CLASS OF ORTHOTROPIC PLATES

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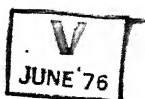
# **VIBRATION OF A CLASS OF ORTHOTROPIC PLATES**

**A Thesis Submitted  
in partial Fulfilment of the Requirements  
for the Degree of  
MASTER OF TECHNOLOGY**

**By  
INDER KRISHEN PANDITTA**

**to the**

**DEPARTMENT OF AERONAUTICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
JANUARY, 1976**



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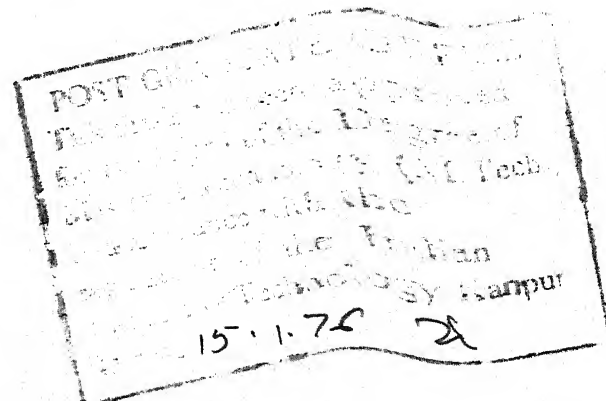
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CERTIFICATE

Certified that the work titled "Vibrations of  
a Class of Orthotropic Plates" has been carried out under  
my supervision and has not been submitted elsewhere for a  
degree.

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## ABSTRACT

The vibrations of orthotropic cantilever and free-free plate have been studied using Rayleigh-Ritz method.

The convergence of Rayleigh-Ritz method and the effect of different plate parameters have been investigated.

Calculations for frequency parameter has been carried out for five ratios of Young's modulus each incorporating five poisson's ratios and ten aspect ratios. These have been tabulated and can be used for design purposes. These design tables contain first two modes of cantilever plate and first mode of free-free plate.

## NOMENCLATURE

$X, Y$	=	Cartesian co-ordinates in the plane of plate
$a$	=	Plate dimension perpendicular to built in edge
$b$	=	Plate dimension parallel to built in edge
$h$	=	Plate thickness
$W$	=	Plate deflection normal to X-Y plane
$\epsilon_x$	=	Normal strain in x-direction
$\epsilon_y$	=	Normal strain in y-direction
$\gamma_{xy}$	=	Shear strain in x-y plane
$\tau_{xy}$	=	Shear stress in x-y plane
$\sigma_x$	=	Normal stress in x-direction
$\sigma_y$	=	Normal stress in y-direction
$E_x$	=	Young's modulus in x-direction
$E_y$	=	Young's modulus in y-direction
$D_i$	=	$E_i / (1 - \nu_x \nu_y)$
$\omega$	=	Circular frequency
$\nu_x$	=	Poisson's ratio in x-direction
$\nu_y$	=	Poisson's ratio in y-direction

## CHAPTER 1

### INTRODUCTION

#### 1.1 Introduction:

The advent of advanced fiber re-inforced composite materials such as boron-epoxy and graphite-epoxy with their high potential weight savings and frequent applications in several fields of technology — Aerospace industry; Civil, Mechanical and Ocean engineering, etc. — have generated the need of sound understanding of the static and dynamic behaviour of anisotropic structures. A number of methods are available for predicting the dynamic response of the systems but the knowledge of natural modes and natural frequencies of the system is required by most of them.

An exact solution of the differential equation of a vibrating plate is known for the case of a rectangular plate which is simply supported along one pair of opposite edges with any conditions at the other two edges. For other combinations of edge conditions the solutions are more complicated, and it has been necessary to resort to various approximate methods — Rayleigh-Ritz being one of them.

While Rayleigh-Ritz method is well known, it has not been used as much as might be expected for plate

vibration problems. There appears to be little published data for the vibrations of rectangular orthotropic plate. This is probably due, at least in part, to the great amount of computational labour which is required both to set up and to solve the necessary equations. The amount of computation involved depends to a large extent upon the set of functions that is used to represent the plate deflections. For these functions some investigators (1,2) have taken series of polynomials while others (3,7) have used combinations of the characteristic functions which define the normal modes of vibration of a uniform beam.

## 1.2 Historical Preview:

In 1966 Laura and others (1) used Galerkins method for calculating natural frequencies of rectangular plate clamped at all edges. First four frequencies were calculated for different  $b/a$  ( $a > b$ ) and were put in a tabular form.

In 1973 Maurizi and Laura (2) made use of simple polynomial approximation for the determination of natural frequencies of clamped orthotropic plate. Galerkins method was used to investigate the effect of the rotation of material elastic axis with respect to natural coordinate system.

Young (3) used Ritz method for calculating frequencies of transverse vibration of an isotropic plate with different boundary conditions. Beam functions were employed to represent the plate deflection.

In 1969, Ashton and Anderson (4) worked on the natural frequencies of the laminated boron-epoxy plate with fully fixed boundary conditions.

Almost simultaneously Ashton (5) published his work on natural frequencies of free rectangular plate laminated of orthotropic plies. His work mostly concentrates on the stability problem. The solutions which illustrate the effect of plies and stacking sequence, have also been presented. The results are not quite agreeing with experimental values.

In 1971, Mohan and Kingsbury (6) used Galerkin's method for plate with different boundary conditions. They used beam functions. Galerkin's method, as they have used it, is good for obtaining mode shapes and natural frequencies of a plate with supported edges but can not be used for a plate with free edges, where the free edge zero bending moment and Krischoff's shear force conditions are not satisfied.

Bassily and Dickson (7) brought out the difference between Galerkin's and Rayleigh-Ritz method clearly and developed a Generalised Galerkin's approach in which the residual boundary forces and moments have been accounted for. But, unfortunately, while carrying out the theoretical analysis they have consistently missed the co-efficients  $C_{11}$  and  $C_{22}$  of  $(\frac{\epsilon_i}{a})^4$  and  $(\frac{\eta_k}{b})^4$  respectively in their equations (3) and (5), hence rendering their numerical results useless.

### 1.3 Present Work:

In the present work Rayleigh-Ritz method has been employed for analysing the vibrating cantilever and free plate, as closed form solutions are almost impossible. In the application of Rayleigh-Ritz method a series in which each term is a product of normal beam functions has been used. For a plate with clamped edges the appropriateness of the normal functions of beam is apparent. Each term in the series satisfies boundary conditions of the plate and the determination of co-efficients by the minimization process brings about an approach to satisfaction of the differential equation. But, when the edges of the plate are free, as in our case, the normal functions of a beam with free ends do not give terms in the series which each

satisfy the free-edge boundary conditions of the plate. The beam functions will have vanishing second and third order derivatives. The satisfaction of the free-edge plate conditions requires non-zero values of corresponding derivatives. The plate boundary conditions thus remain to be satisfied by the series as a sum. The minimization process is relied on for this as bringing about an approach to the satisfaction of natural boundary conditions.

A generalised computer programme has been worked out in which (a) number of materials (b) number of terms in the series of beam functions (c) aspect ratios with fixed step size (d) modal frequency numbers (e) symmetry, antisymmetry of the problem involved can be controlled by merely changing the appropriate data cards.

With the help of this programme convergence of Rayleigh-Ritz method; effect of aspect ratio, Poisson's ratio and ratio of Young's modulus of the plate have been investigated. Few design tables have also been presented.

## CHAPTER 2

### THEORETICAL ANALYSIS

#### 2.1 Introduction:

In this chapter equations for vibrations of orthotropic plate are developed and the method of solution is discussed.

#### 2.2 General Theory:

The approximate theory for bending of a plate based on Krischoff's hypothesis — rectilinear sections which in the undeformed plate were normal to the middle surface remain rectilinear and normal to the bent middle surface (OR  $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$ ). And normal stress acting on planes parallel to mid surface are small compared to other stresses and can be neglected (OR  $\sigma_z = 0$ ) — leads to the following strain-displacement relations:

$$\epsilon_x = \frac{\partial u}{\partial x} = -z \frac{\partial^2 w}{\partial x^2} \quad . . . . 2.1a$$

$$\epsilon_y = \frac{\partial v}{\partial y} = -z \frac{\partial^2 w}{\partial y^2} \quad . . . . 2.1b$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z \frac{\partial^2 w}{\partial x \partial y} \quad . . . . 2.1c$$

which are same as for an isotropic case. The deviation from isotropicity shows up only in stress-strain relations which can be written as:

$$\sigma_x = \frac{E_x}{1 - \nu_x \nu_y} (\epsilon_x + \nu_y \epsilon_y) \quad \dots .2.2a$$

$$\sigma_y = \frac{E_y}{1 - \nu_x \nu_y} (\epsilon_y + \nu_x \epsilon_x) \quad \dots .2.2b$$

$$\tau_{xy} = \frac{\sqrt{E_x E_y}}{2(1 + \sqrt{\nu_x \nu_y})} \gamma_{xy} \quad \dots .2.2c$$

Potential energy  $V$  for a plate is given by

$$V = \frac{1}{2} \iiint_{\tau} (\sigma_x \epsilon_x + \sigma_y \epsilon_y + \tau_{xy} \gamma_{xy}) d\tau \quad \dots .2.3$$

making use of equations (2.1) and (2.2) the above equation reduces to

$$\begin{aligned} V = \frac{1}{2} \iiint_{\tau} & \left[ \frac{E_x z^2}{(1 - \nu_x \nu_y)} \left( \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) \frac{\partial^2 w}{\partial x^2} \right. \\ & + \frac{E_y z^2}{(1 - \nu_x \nu_y)} \left( \frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2} \right) \frac{\partial^2 w}{\partial y^2} \\ & \left. + \frac{2 \sqrt{E_x E_y}}{1 + \sqrt{\nu_x \nu_y}} z^2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] d\tau \quad \dots .2.4 \end{aligned}$$

Integrating this with respect to  $z$  from  $-h/2$  to  $h/2$  we get

$$V = + \frac{h^3}{24} \iint_A \left[ D_x \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + 2 D_{xy} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + D_y \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + 2 \sqrt{D_x D_y} (1 - \nu_x \nu_y) \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] dx dy \quad \dots .2.5$$

The kinetic energy  $T$  of the system is given by

$$T = \frac{1}{2} \iint_A \rho h \left( \frac{\partial w}{\partial t} \right)^2 dx dy \quad \dots .2.6$$

Using the fact that natural modes execute harmonic motion we can write

$$U = T + V = V - \frac{\rho h \omega^2}{2} \iint w^2 dx dy \quad \dots .2.7$$

$$\text{where} \quad w = W(x, y) e^{i\omega t} \quad \dots .2.8$$

The function  $W(x, y)$  will be interpolated as beam functions or

$$W(x, y) = \sum_m \sum_n A_{mn} \phi_m \left( \frac{\epsilon_m x}{a} \right) \psi_n \left( \frac{\eta_n y}{b} \right) \quad \dots .2.9$$

where  $\phi_m$  and  $\psi_n$  are beam functions in  $x$  and  $y$  directions respectively.

When  $W(x,y)$  as given by equation (2.9) is substituted in equation (2.7), the right hand side becomes function of the co-efficients  $A_{mn}$ . This is minimized by taking the partial derivative with respect to each coefficient and equating to zero. Thus we arrive at a set of equations each of which has the form

$$\frac{\partial V}{\partial A_{ik}} - \frac{\omega^2 \rho h}{2} \frac{\partial}{\partial A_{ik}} \iint W^2 dx dy = 0 \quad \dots 2.10$$

where  $A_{ik}$  is any one of the coefficients  $A_{mn}$ . Equation (2.10) represents a system of linear homogeneous equations in the unknowns  $A_{mn}$ . The natural frequencies  $\omega_1, \omega_2, \dots$  are determined from the condition that the determinant of the system must vanish.

### 2.3 Characteristic Functions for Vibrating Beam:

The different types of beams will be identified by a compound objective which describes the end conditions. Thus a "clamped-clamped" beam is one which is rigidly clamped at both ends; a "clamped-free" beam is clamped at the end  $x = 0$  and free at the end  $x = l$ ; a "free-free" beam is free at both ends.

For each type of beam there is an infinite number of normal modes in which the beam can vibrate laterally.

The method of determining the set of characteristic functions which define the normal modes for any type of beam is given in standard references such as (8) and (9). The characteristic functions for the two types of beams used in present work are as follows.

#### Clamped-Free Beam

$$X_r = \cosh \frac{\epsilon_r x}{l} - \cos \frac{\epsilon_r x}{l} - \alpha_r \left( \sinh \frac{\epsilon_r x}{l} - \sin \frac{\epsilon_r x}{l} \right)$$

$$r = 1, 2, 3, \dots \quad \dots 2.11$$

#### Free-Free Beam

$$X_1 = 1 \quad \dots 2.12a$$

$$X_2 = \sqrt{3}(1 - 2x/l) \quad \dots 2.12b$$

$$X_r = \cosh \frac{\epsilon_r x}{l} + \cos \frac{\epsilon_r x}{l} - \alpha_r \left( \sinh \frac{\epsilon_r x}{l} + \sin \frac{\epsilon_r x}{l} \right)$$

$$(r = 3, 4, 5, \dots) \quad \dots 2.12c$$

Each expression defines an infinite set of functions. The numerical values of  $\alpha_r$  and  $\epsilon_r$  for each set of functions are given in appendix.

Tables of values of these functions is given in (10) to five decimal places and at intervals of the argument  $x/l = 0.02$ .

Equation (2.12c) is the usual expression for the characteristic functions of a free-free beam, when  $r = 3$  we have the first mode of free vibration. The functions  $x_1$  and  $x_2$  represent a rigid body translation and rotation and are included in order to obtain a complete orthogonal set.

The boundary conditions satisfied by the functions in each set are the same as the end conditions of the corresponding beam. That is, for the clamped-free functions  $X_r = \frac{dX_r}{dx} = 0$  at  $x = 0$  and  $d^2X_r/dx^2 = d^3X_r/dx^3 = 0$  at  $x = 1$ ; for the free-free functions

$$\frac{d^2X_r}{dx^2} = \frac{d^3X_r}{dx^3} = 0 \quad \text{at } x = 0 \text{ and } x = 1$$

Each of the characteristic functions except those of equations (2.12a) and (2.12b) satisfies the differential equation  $d^4X_r/dx^4 = \epsilon_r^4(X_r/l^4)$ . Each set of the functions is orthogonal in the interval 0 to 1, that is, for any two functions  $X_r$  and  $X_s$  in the same set, the following equations hold

$$\frac{1}{l} \int_0^1 X_r X_s dx = \delta_{rs} \quad \dots \dots \dots 2.13$$

$$\begin{aligned} \text{where } \delta_{rs} &= 1 & \text{for } r &= s \\ &= 0 & \text{for } r &\neq s \end{aligned}$$

The second derivatives of the function in each set are also orthogonal and satisfy the relations

$$\int \frac{d^2 X_r}{dx^2} \frac{d^2 X_s}{dx^2} dx = \frac{\epsilon_r^4}{l^3} \quad (\text{for } r = s)$$

$$= 0 \quad (\text{for } r \neq s) \quad \dots 2.14$$

With the exception of  $X_1$  and  $X_2$  for the free-free functions, equations (2.12a) and (2.12b), for which

$$\int_0^1 \left( \frac{d^2 X_1}{dx^2} \right)^2 dx = \int_0^1 \left( \frac{d^2 X_2}{dx^2} \right)^2 dx = 0 \quad \dots 2.15$$

In addition to the integrals defined by equations (2.13) and (2.14), it is necessary to evaluate

$$\int_0^1 X_r \frac{d^2 X_s}{dx^2} dx \quad \text{and} \quad \int_0^1 \frac{dX_r}{dx} \frac{dX_s}{dx} dx.$$

When using these functions in Raleigh-Ritz method. Values of these integrals have been tabulated in appendix.

Using Equations (2.9) and (2.5), and taking into account the orthogonality relations, equations (2.13) and (2.14), the set of equations (2.10) can be reduced to the form

$$\sum_m \sum_n [C_{mn}^{(ik)} - \lambda^2 \delta_{mn}] A_{mn} = 0 \quad \dots .2.16$$

where

$$C_{mn}^{(ik)} = \nu_y (E_{mi} F_{kn} + E_{im} F_{nk}) + 2 \sqrt{\frac{D_y}{D_x}} (1 - \sqrt{\nu_x \nu_y}) H_{im} K_{kn} \\ + \left( \frac{\epsilon_i^4}{\alpha^2} + \frac{D_y}{D_x} \alpha^2 \eta_k^2 \right) \delta_{mn}$$

$$\lambda^2 = \frac{12 \rho \omega^2 a^2 b^2}{h^2 D_x}$$

$\alpha = a/b = \text{Aspect ratio}$

$$H_{im} = a \int_0^a \left[ \frac{d \phi_i(x)}{dx} \frac{d \phi_m(x)}{dx} \right] dx$$

$$K_{kn} = b \int_0^b \left[ \frac{d \psi_k(y)}{dy} \frac{d \psi_n(y)}{dy} \right] dy$$

$$E_{im} = a \int_0^a \left[ \phi_i(x) \frac{d^2 \phi_m(x)}{dx^2} \right] dx$$

$$F_{kn} = b \int_0^b \left[ \psi_k(y) \frac{d^2 \psi_n(y)}{dy^2} \right] dy$$

and  $\epsilon_i$  and  $\eta_k$  are the eigenvalues associated with the beam modes  $\phi_i(\epsilon_i x/a)$  and  $\psi_k(\eta_k y/b)$  respectively. (Numerical values for the quantities  $\epsilon_i$ ,  $\eta_k$ ,  $E_{im}$ ,  $F_{kn}$ ,  $H_{im}$ ,  $K_{kn}$  are tabulated by Young (3) in his paper on isotropic plates and have been reproduced in Appendix II).

#### 2.4 Method for Solution:

There will be one equation for each m.n combination of ik. The characteristic values for  $\lambda$  are found from the condition that determinant of this system of equations must vanish. If there are more than three or four equations in the system, the mathematical labour of expanding the determinant and solving for the roots of the polynomial in  $\lambda$  is prohibitive. In such cases it is expedient to solve for  $\lambda$  by one of the known iterative procedures. One of the advantages of using beam functions is that the diagonal terms in the determinant are large compared to the others and as a result the characteristic values and modes can be found by simple iteration procedure.

For a cantilever plate using 24 term series with  $m = 1, 2, 3, 4$  and  $n = 1, 2, 3, 4, 5, 6$ ; equation (2.16) gives us 24 equation which can be split up in two groups of 12 equations each. One of these groups include only

$n = 1, 3, 5$  and represent deflections which are symmetrical about the line  $y = b/2$ . The other group includes only  $n = 2, 4, 6$  and represents deflections which are antisymmetric with respect to the line  $y = b/2$ .

For free-free plate the system of equation can be divided into four groups by combination of  $m$  and  $n$  as odd or even.

Trial vector for  $A_{mn}$  is chosen and the iterations are carried out till we get convergence to the proper limit.

### CHAPTER 3

#### PRESENTATION OF RESULTS

##### 3.1 General:

All the results obtained by the present analysis have been tabulated and some of them have been plotted. The results have been compared with the available ones wherever it was possible and the interpretation of the results has been carried out.

##### 3.2 Comparison of Results:

In the present work calculations were carried out for five different materials out of which one falls in the category of isotropic materials. The properties of different materials which were selected for the present work are given in Table 3.2.1.

Nature of material	$E_x$ (Ksc)	$E_y$ (Ksc)	$\nu_x$	$\nu_y$	$E_y/E_x$
Plywood	$1 \times 10^5$	$0.5 \times 10^5$	0.05	0.025	0.5
Plywood	$1 \times 10^5$	$0.05 \times 10^5$	0.2	0.01	0.05
Epoxy resin	$2.8 \times 10^5$	$0.224 \times 10^6$	0.2	0.016	0.08
Graphite-epoxy	-	-	0.24	0.0165	0.06875
Isotropic	-	-	0.3	0.3	1.0

TABLE 3.2.1

Table 3.2.2 compares the values for frequency parameter obtained by Young (3) for square cantilever isotropic plate with the values obtained by present analysis.

$\frac{\omega}{\sqrt{D/\rho h a^4}}$	1 <sup>st</sup> mode	2 <sup>nd</sup> mode	3 <sup>rd</sup> mode	4 <sup>th</sup> mode	5 <sup>th</sup> mode
Young	3.494	8.547	21.44	27.46	31.17
Present values	3.4937	8.5441	21.44	27.4562	31.17

TABLE 3.2.2

The values were obtained by taking 18 terms in equation (2.16). As can be seen from the table the values obtained by present analysis are in excellent agreement with the values obtained by Young (3).

Barton (11) calculated the values of frequency parameter  $\bar{\lambda} (= \frac{\omega}{\sqrt{D/\rho h a^4}})$  of an isotropic plate with different aspect ratios ( $= \frac{a}{b}$  ;  $a > b$ ). These values are tabulated in Table 3.2.3. Comparing these values with the values obtained by present analysis, given in Table 3.2.4 after changing  $\lambda$  to  $\bar{\lambda}$ , we find that the values are same except in second mode in which present values are slightly lower.

$\bar{\lambda}$  Obtained by Barton for Isotropic Cantilever Plate

a/b	Mode Numbers				
	1	2	3	4	5
0.5	3.508	5.372	21.96	10.26	24.85
1.0	3.494	8.547	21.44	27.46	31.17
2.0	3.472	14.93	21.61	94.49	48.71
5.0	3.45	34.73	21.52	563.9	105.9

TABLE 3.2.3

$\bar{\lambda}$  Obtained by Present Analysis for Isotropic Cantilever Plate

a/b	Mode Numbers				
	1	2	3	4	5
0.5	3.508	5.372	21.96	10.26	24.85
1.0	3.494	8.544	21.44	27.46	31.17
2.0	3.472	14.92	21.61	94.48	48.71
5.0	3.45	34.71	21.52	563.9	105.9

TABLE 3.2.4

Table 3.2.5 compares the results given by Basily and Dickson (7) and Mohan and Kingsbury (6). Present values are higher than the values given by both of them. It is because Mohan and Kingsbury used Galerkins method without modifying it for the free edges whereas Basely and Dickson used Ritz method but their frequency equation is wrong as has been pointed out earlier.

$\lambda$  for Cantilever Orthotropic Plate

$$E_y/E_x = 0.06875 \quad \text{and} \quad \nu_x = 0.24$$

r	s	Basily & Dickson (Ritz)		Mohan & Kingsbury	Present Analysis (Raleigh-Ritz)
		4x4	4x5	Galerkin (4x4)	5x6
0	0	3.514	3.515	3.516	3.5157
0	1	4.191	4.191	3.516	5.9016
0	2	8.205	8.181	6.5	11.3686
0	3	17.76	17.75	16.4	21.0445
1	0	22.03	22.03	22.03	22.03
1	2	26.14	25.96	23.1	34.3139
1	1	22.87	22.87	22.03	25.4860
0	4	-	32.98	-	34.3149
1	3	33.34	33.33	28.6	47.0021

TABLE 3.2.5

### 3.3 Summary and Conclusions:

All the results have been tabulated and are presented. Figures have been drawn to bring out the dependence of frequency parameter on different plate parameters.

#### 3.3.1 Convergence of Rayleigh-Ritz Method

Calculations were carried out with 18 (T. Nos. 1.00, 1.10, 1.20 and 1.30), 24 (T. Nos. 1.01, 1.11, 1.21 and 1.31), 30 (T. Nos. 1.02, 1.12, 1.22 and 1.32) number of terms in the interpolating function. The convergence of Rayleigh-Ritz method with respect to number of terms in the interpolating function is very good as can be observed by comparing the above said tables.

Convergence of Rayleigh-Ritz method is fairly good except for certain frequencies which did not converge even after iterating them for half an hour on 7044 computer. The frequencies which did not converge were:

T. No.	Mode	Aspect ratio
2.00	(1,1)	0.60
2.20	(1,0)	0.90
2.30	(1,1)	0.30

TABLE 3.3.1

# CANTILEVER PLATE

⊗ Mode (0,0)

○ Mode (0,1)

No	RYX	$\gamma_x$
1-1	1.00	0.3
2-2	0.50	0.05
3-3	0.08	0.20
4-4	0.06	0.24
5-5	0.05	0.20

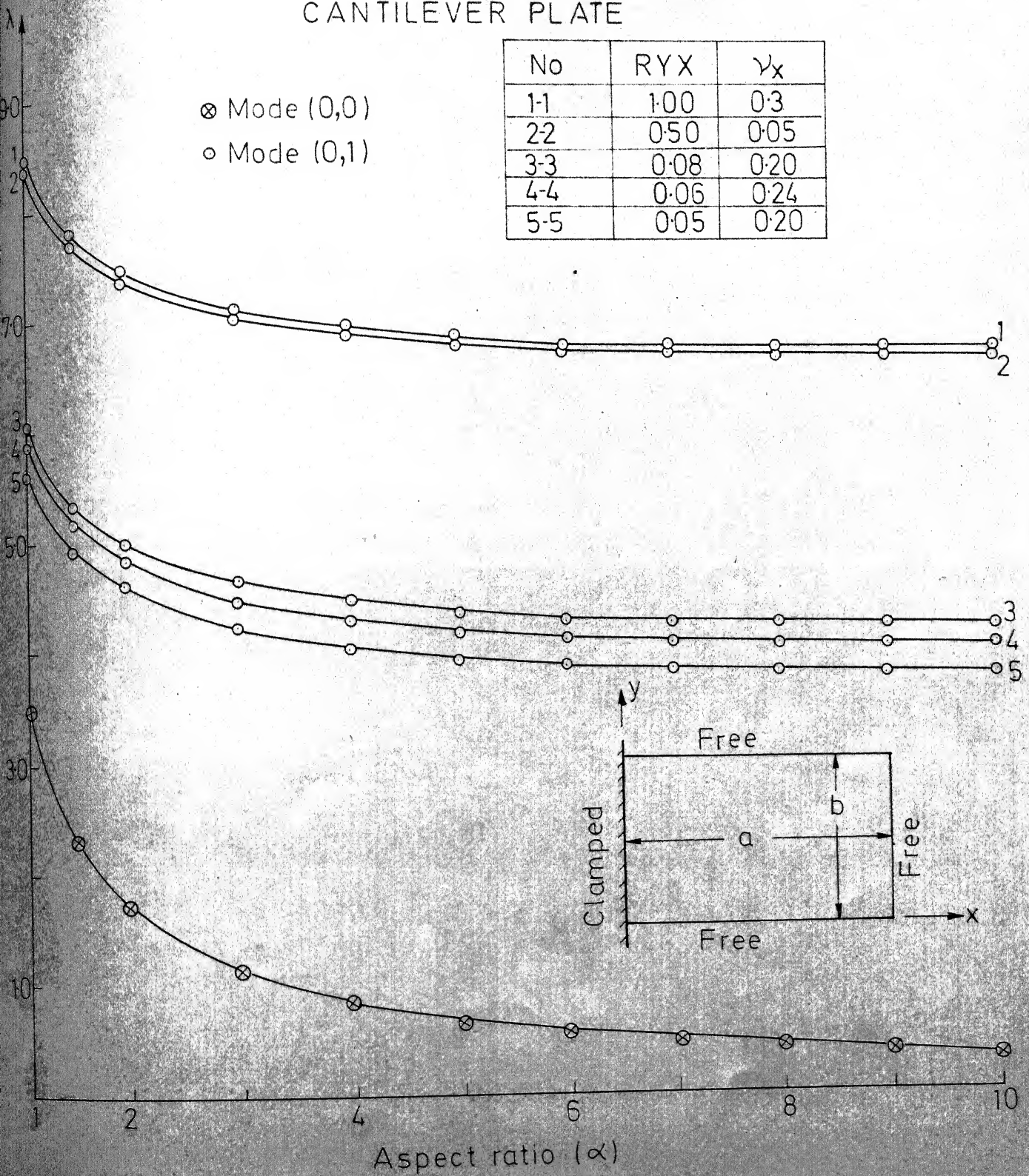


FIG. 1

# CANTILEVER PLATE

⊗ Mode (1,1)  
○ Mode (1,0)

No	RYX	$\gamma_x$
1-1	1.00	0.30
2-2	0.50	0.05
3-3	0.08	0.20
4-4	0.06	0.24
5-5	0.05	0.20

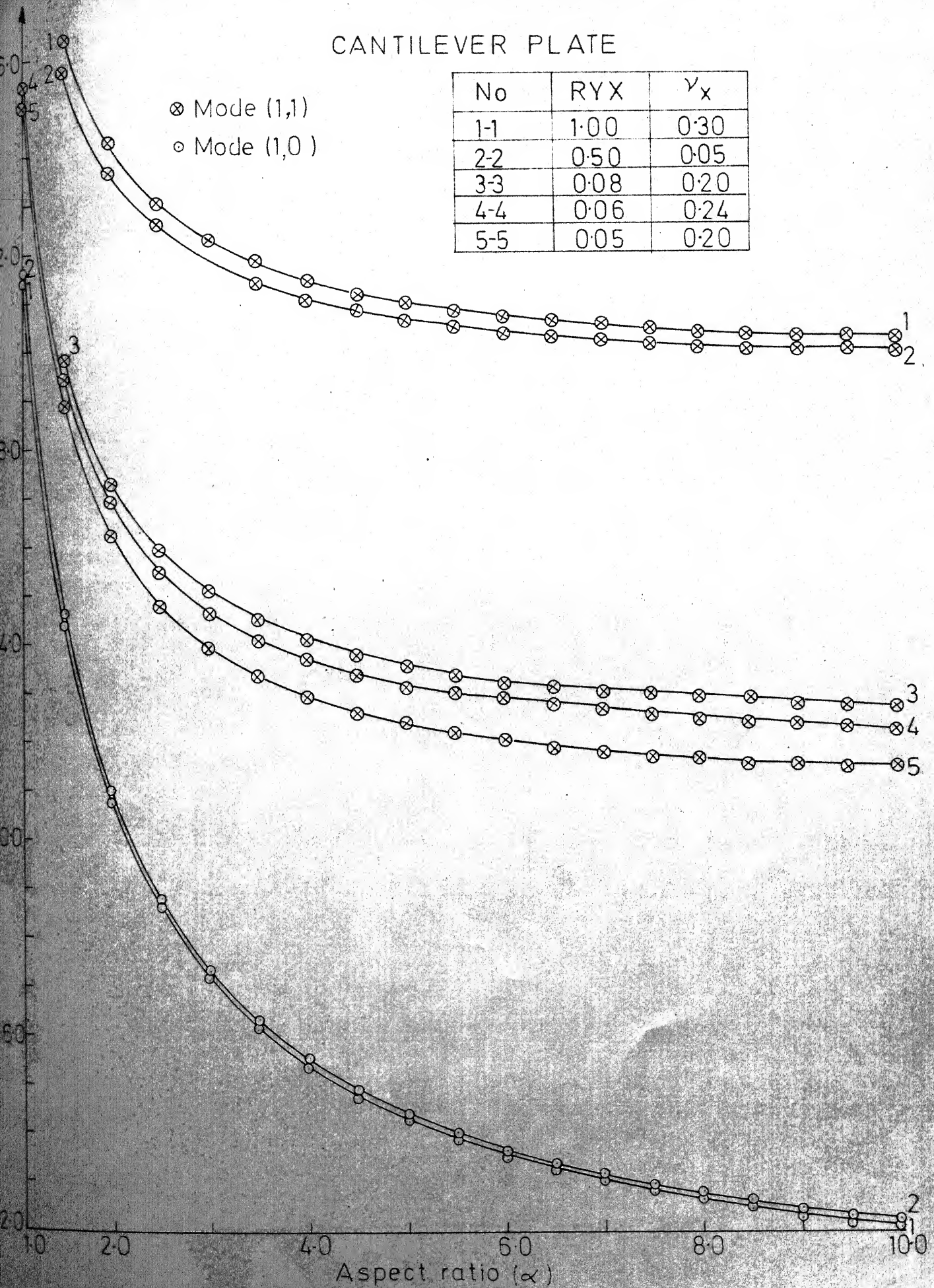


FIG. 2

# CANTILEVER PLATE MODE (0,2)

No	RYX	$\nu_x$
1-1	1.00	0.30
2-2	0.50	0.05
3-3	0.08	0.20
4-4	0.06	0.24
5-5	0.05	0.20

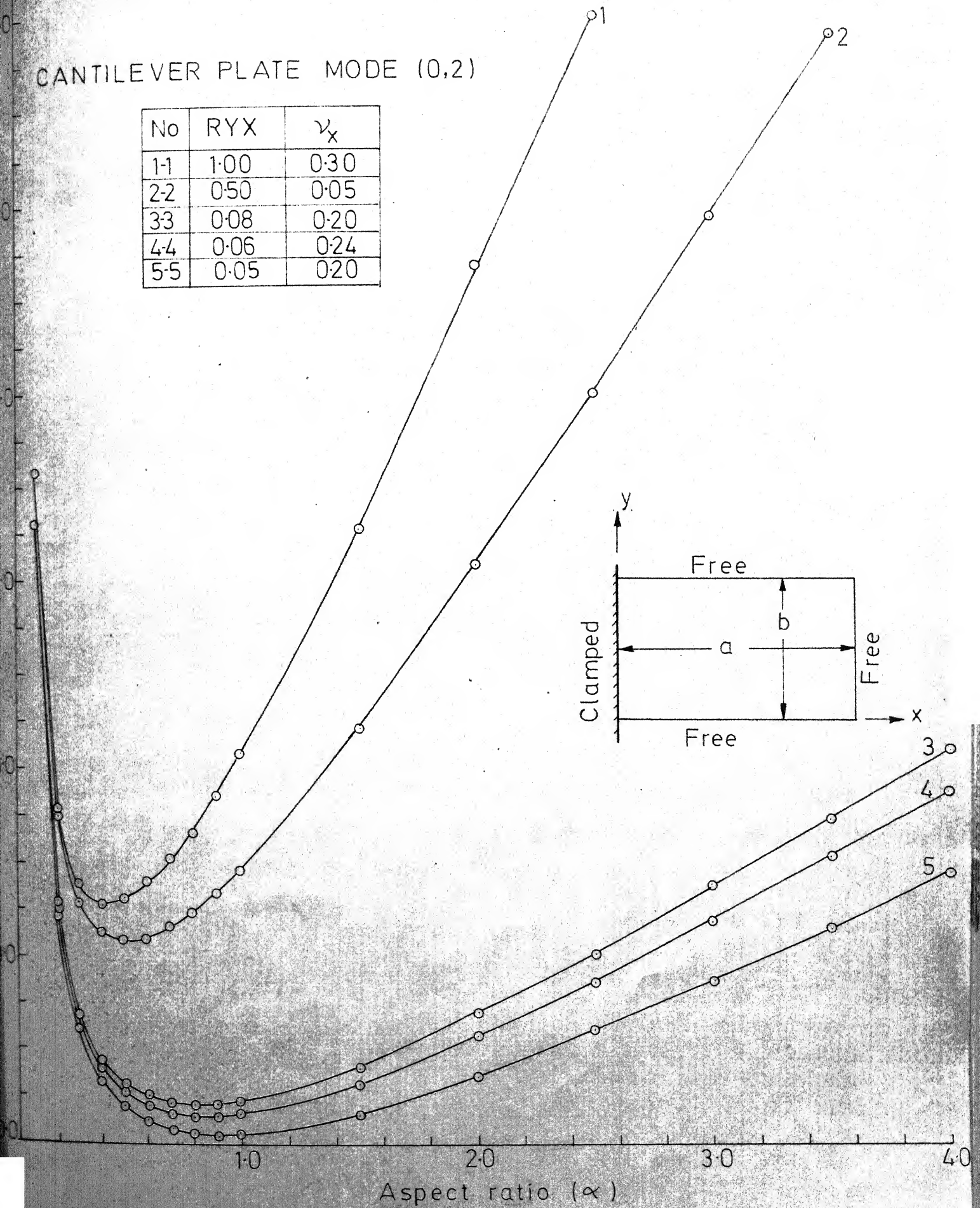


FIG. 3

It seems that after few iterations either the convergence rate becomes poor or the iterations itself start oscillating.

### 3.3.2 Effect of Aspect Ratio on Frequency Parameters

Frequency parameter  $\lambda$  decreases for modes (0,0), (0,1), (1,0) and (1,1) for increasing aspect ratio (Figs. 1 and 2). The rate of decrease for the first two modes is lower than the other two. The decrease in the frequency parameter should be expected as the aspect ratio increases because the effective stiffness of the plate decreases.

For mode (0,2) it shows a minimum and hence increases after some point as aspect ratio increases (Fig. 3). This increase is not in any case contradicting to the statement given earlier because decrease in frequency does not necessarily mean a decrease in frequency parameter. Frequency parameter is itself a function of aspect ratio and hence an increase in aspect ratio and decrease in frequency can result in an increase in the frequency parameter. If we change the frequency parameter from  $\lambda^2 = 12\rho\omega^2 a^2 b^2 / h^2 D_x$  to  $\bar{\lambda}^2 = 12\rho\omega^2 b^4 / h^2 D_x$  then the frequency parameter shows a decreasing trend as is shown in Table 3.3.2.

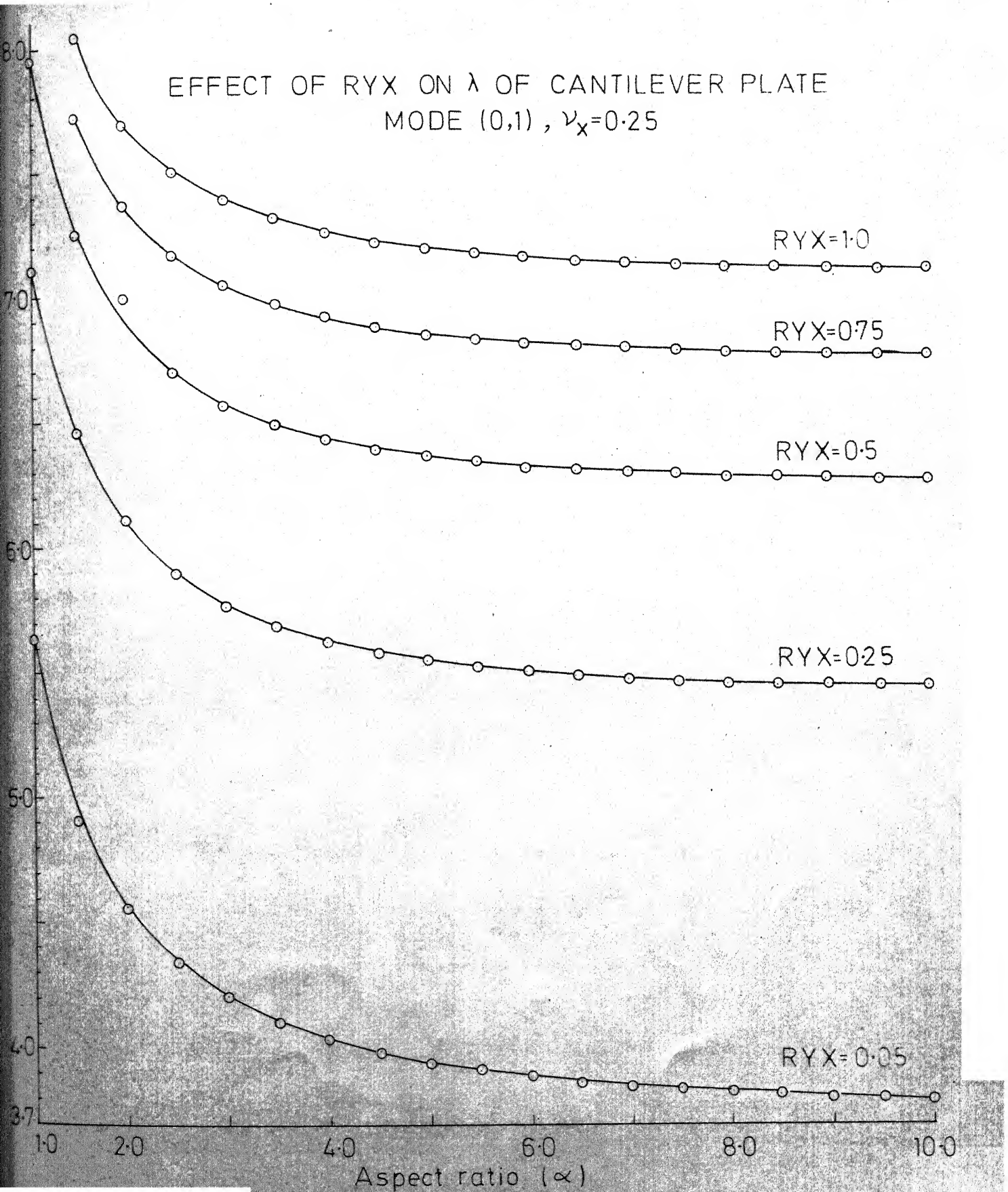


FIG.4

EFFECT OF  $R_{YX}$  ON  $\lambda$  OF CANTILEVER PLATE  
MODE (1,1) ,  $\nu = 0.25$

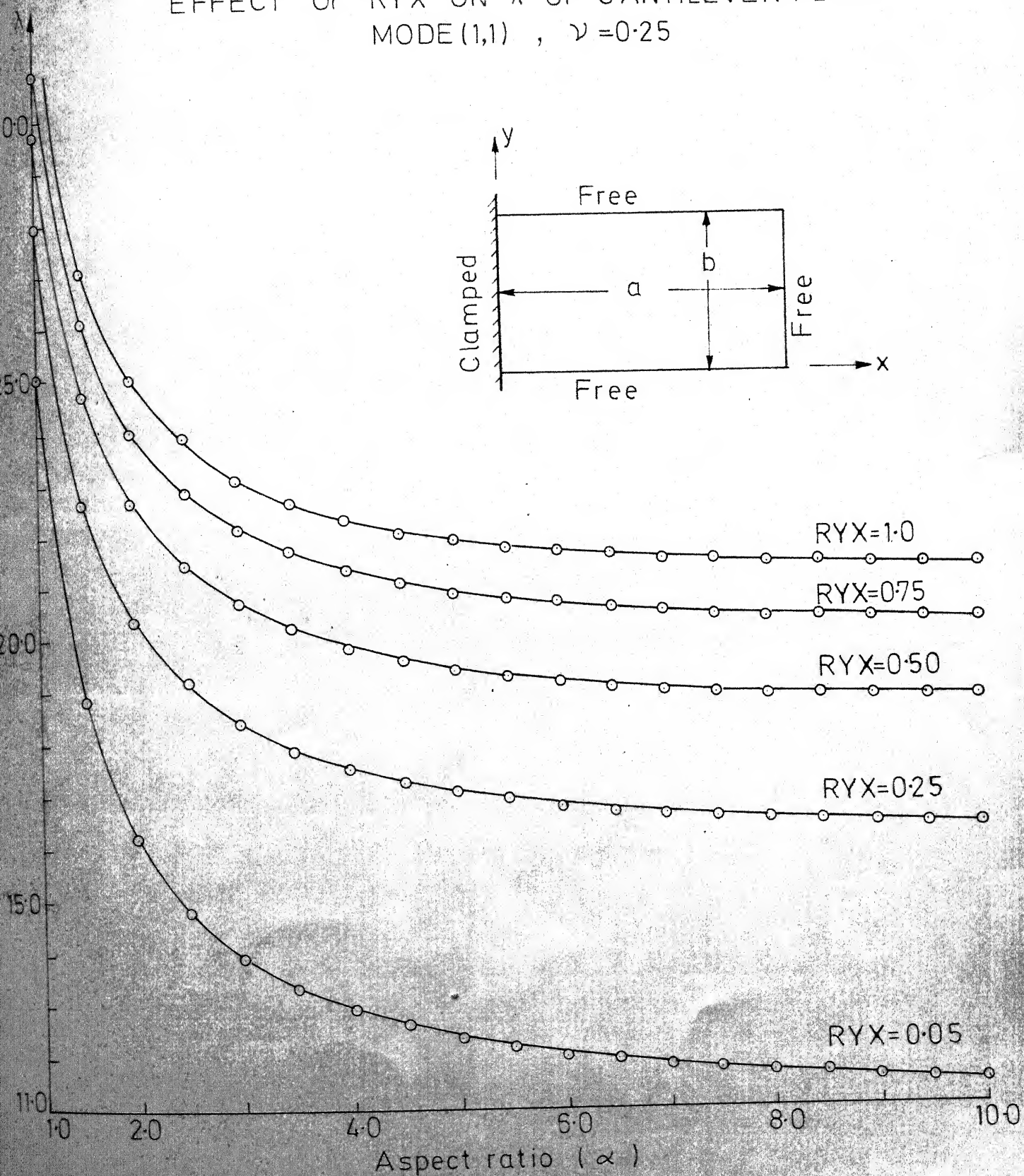


FIG. 5

Aspect ratio	1.00	2.00	5.00	10.00
$\bar{\lambda}$	10.5042	6.4844	5.2397	5.0618

TABLE 3.3.2

Here increase in aspect ratio and decrease in frequency parameter can be interpreted as increase in dimension  $a$  i.e. length of cantilever side, decreases frequency which is expected.

### 3.3.3 Effect of $E_y/E_x$ on frequency parameter

Referring to Figs. 4 and 5 it can be seen that increase in  $RYX (= E_y/E_x)$  brings about an increase in frequency parameter, as should be expected, for antisymmetric modes. For symmetric modes the frequency parameter does not show significant dependence on  $RYX$  as can be seen from Tables 4.00 and 4.20.

### 3.3.4 Effect of Poissons Ratio on frequency parameter

For symmetrical modes frequency parameter is practically independent of poisons ratio, as can be observed from Tables 3.00 and 3.20. The maximum change being 0.2%.

For antisymmetric modes the frequency parameter changes merely by 4% (maximum). See Tables 3.10 and 3.30.

### 3.3.5 Discussion on Free Plate

Frequency parameter  $\lambda$  has been calculated for first five modes of the material with  $E_y/E_x = 0.06875$  and Poissons ratio = 0.24 (Table 5.0).

First three modes have zero frequencies and these represent rigid body translation and rigid body rotation about  $x = a/2$  and  $y = b/2$  line.

Mode (1,1) shows a minimum whereas mode (2,2) increases steadily which is not unexpected because if  $\lambda$  is changed to  $\bar{\lambda}$ , both these modes show a decreasing trend.

Referring to design tables for free plate one can observe that effect of RYX is pronounced. As RYX increases  $\lambda$  also increases and the effect of poissions ratio on  $\lambda$  also becomes significant.

#### 4 Scope for Further Research:

Following areas can be explored further.

- . Use of Generalized Galerkins approach to solve such problems would give more insight even though it may involve great amount of computational labour.

2. Convergence rate of Raleigh-Ritz method may be studied to investigate the convergence of such frequency parameters as given in Table 3.3.1.
3. Effect of non-linear vibrations on the frequency parameter can be studied.
4. Vibrations of orthotropic plates with other boundary conditions can be studied.

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# A P P E N D I X - I

AEGX 4, TIME008, PAGE005, NAME INDER KRISHN PANDITTA . 5.3

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TC

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*****
*
*   THIS PROGRAM CAN EVALUATE EIGENVALUES AND EIGENVECTORS OF A PLATE
*   WITH ANY COMBINATION OF CANTILEVER AND FREE EDGES.
*
*   NAR  =NUMBER OF ASPECT RATIOS FOR WHICH RESULT IS REQUIRED. ASPECT RATIO
*   STARTS FROM 1.0 AND IS INCREMENTED BY DELAR.
*   NESAS=LEFFECTIVE SYMMTRY ANTISYMMTRY.
*   NX   =NUMBER OF TERMS IN X-DIRECTION.
*   NY   =NUMBER OF TERMS IN Y-DIRECTION.
*   NEX  =NUMBER OF EFFECTIVE TERMS IN X-DIRECTION.
*   NEY  =NUMBER OF EFFECTIVE TERMS IN Y-DIRECTION.
*   NMF  =NUMBER OF MATERIALS FOR WHICH RESULT IS REQUIRED.
*   NMF  =NUMBER OF MODAL FREQUENCIES.
*   NSMF =NUMBER OF MODAL FREQUENCY FROM WHICH YOU WANT TO START.
*   NJMF =INTERVAL IN THE MODAL FREQUENCIES.
*   AR   =ASPECT RATIO.
*   RYX  =RATIO OF Y-DIRECTIONAL YOUNG&S MODULAS TO X-DIRECTIONAL YOUNG&S
*   MODULAS.
*   PX   =POISON&S RATIO IN X-DIRECTION.
*   PY   =POISON&S RATIO IN Y-DIRECTION.
*   EP   =ACCURACY FOR EIGENVALUES AND EIGENVECTORS.
*   ICONV=MAXIMUM ITERATIONS ALLOWED FOR CONVERGENCE.
*
*   X-SYMMTRY          NSASY=1 AND 2   NUJY=1
*   Y-SYMMTRY          NSASY=1 AND 2   NUJY=1
*
*   IF THERE IS NO SYMMTRY      NSASX=1      NUJX=1
*   AND                          NSASY=1      NUJY=1
*****
```

```
DIMENSIONA(20),AA(20),B(20,20),C(20,20),D(20,20),E(20,20),F(20,20),
1EY(20),F(20,20),H(20,20),SMM(20),RYX(20),PX(20),PY(20)
```

```
READ 90,((E(I,J),I=1,5),J=1,5)
READ 90,((H(I,J),I=1,5),J=1,5)
READ 90,((B(I,J),I=1,7),J=1,7)
READ 90,((F(I,J),I=1,7),J=1,7)
READ 90, (EX(I),I=1,5)
READ 90, (EY(I),I=1,7)
```

```
READ 90, EP
```

READ 992,NAR,NSAS,NX,NY,NXY,NBY,NMAT,ICONV

READ 993,NSMF,NMF,NJMF

READ 994,(RYX(I),PX(I),I=1,NMAT),DELAP

FORMAT(10I2)

FORMAT(2E15.8)

FORMAT(8(F10.5))

DO 991 III=1,NSAS

READ 992,NSASX,NSASY,NJX,NJY

PRINT 6,NSMF,NMF,NJMF

FORMAT(/10X,\*THE MODAL FREQUENCIES CALCULATED BELOW ARE FROM 10.\*  
1,I2,\* TO NO.\*,I2,\* IN STEPS OF\*,I2//)

PRINT 5,NSASX,NSASY,NJX,NJY,NX,NY

FORMAT(10X,\*NSASX=\*,I2,10X,\*NSASY=\*,I2,10X,\*NJX=\*,I2,10X,\*NJY=\*,  
1I2,10X,\*NX=\*,I2,10X,\*NY=\*,I2//)

DO 999 IMAT=1,NMAT

PY(IMAT)=PX(IMAT)\*RYX(IMAT)

PRINT 95,RYX(IMAT),PX(IMAT)

FORMAT(55X,60(1H\*)/43X,\*RYX=\*,E15.8,10X,\*PX=\*,E15.8/10X,60(1H\*))

AR=0.1

DO 770 IAR =1,NAR

PRINT 772, AR

FORMAT(50X,\*THE FOLLOWING ARE THE RESULTS FOR A CANTILEVER PLATE  
1F ASPECT RATIO=\*,F6.2)

FROM HERE CALCULATION FOR NXY BY NXY MATRIX, USING BEM-FUNCTIONS, STARTS.

J=J

DO 200 I=NSASX,NX,NJX

DO 200 K=NSASY,NY,NJY

L=1

DO 100 M=NSASX,NX,NJX

DO 100 N=NSASY,NY,NJY

IF(M.EQ.I.AND.N.EQ.K) GO TO 110

GO TO 120

C(M,N)=PY(IMAT)\*(E(M,I)\*F(K,N)+E(I,M)\*F(N,K))+2.\*SQRT(RYX(IMAT))\*  
1(1.-SQRT(PX(IMAT)\*PY(IMAT)))\*H(I,M)\*B(K,N)+(PX(I)\*\*4)/AR\*\*2+(RY(K)  
2\*\*4)\*(AR\*\*2)\*RYX(IMAT)

GO TO 101

C(M,N)=PY(IMAT)\*(E(M,I)\*F(K,N)+E(I,M)\*F(N,K))+2.\*SQRT(RYX(IMAT))\*  
1(1.-SQRT(PX(IMAT)\*PY(IMAT)))\*H(I,M)\*B(K,N)

```

D(J,L)=C(M,N)
L=L+1
J=J+1
NXY=NX*NEY
DO 300 I=1,NXY
DO 300 J=1,NXY
C(I,J)=D(I,J)

```

FROM HERE CALCULATIONS FOR EIGENVALUES AND EIGENVECTORS START.

```

DO 608 NF=NSMF,NMF,NJMF
W=0.0
FF=0.0
DO 402 J=1,NXY
AA(J)=0.0
A(J)=0.0
A(NF)=1.0
AA(NF)=1.0
IJ=1
CONTINUE
DO 500 I=1,NXY
SUM=0.0
DO 400 J=1,NXY
IF(I.EQ.J) GO TO 91
GO TO 92
CONTINUE
IF(I.EQ.NF.AND.J.EQ.NF) GO TO 92
GO TO 400

SUM=SUM-C(I,J)*A(J)
CONTINUE
IF(I.EQ.NF) GO TO 403
GO TO 400
W=-SUM
GO TO 500
A(I)=SUM/(C(I,I)-W)
CONTINUE
DO 600 J=1,NXY
IF(ABS((A(J)-AA(J))/A(J)).LE.EP.AND.ABS((W-FF)/W).LE.EP) GO TO 600
DO 602 I=1,NXY
AA(I)=A(I)
FF=W
IJ=IJ+1
IF(IJ.GT.ICONV) GO TO 600
GO TO 603
CONTINUE
WW=W*0.5
PRINT 700,WW,(A(K),K=1,8),IJ,(A(K),K=9,NXY)
FORMAT(/1X,9F14.4/7X,I3,5X,8F14.4)
PRINT 505

```



RYE= 0.05000

PRECISIONS RATIO= 0.10

NUMBER OF TERMS IN SERIES=18

PLOT	MODES				
	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00	1.5159	3.6592	22.0128	10.7042	23.0417
1.50	2.3425	4.9302	14.6920	11.3833	18.9743
2.00	1.7178	4.3812	11.0101	12.9688	16.2571
2.50	1.4061	4.3707	8.8103	14.8828	14.8275
3.00	1.1718	4.2713	7.3420	16.9834	13.9809
3.50	1.0044	4.1283	6.2532	19.2147	13.4125
4.00	0.8788	4.0581	5.5064	21.4960	12.9957
4.50	0.7831	4.0059	4.8946	23.8298	12.6957
5.00	0.7030	3.9661	4.4051	26.2194	12.4575
5.50	0.6391	3.9395	4.0046	28.5914	12.2730
6.00	0.5858	3.9194	3.6708	31.0026	12.1247
6.50	0.5407	3.8946	3.3884	33.4277	12.0029
7.00	0.5021	3.8783	3.1464	35.8638	11.9041
7.50	0.4686	3.8623	2.9366	38.3086	11.8208
8.00	0.4393	3.8513	2.7530	40.7606	11.7506
8.50	0.4135	3.8421	2.5911	43.2184	11.6909
9.00	0.3905	3.8343	2.4471	45.6811	11.6398
9.50	0.3700	3.8277	2.3183	48.1478	11.5957
10.00	0.3515	3.8219	2.2024	50.6180	11.5574

TABLE NO.= 1.00

RYK= 0.05000

POISONS RATIO= 0.20

NUMBER OF TERMS IN SERIES=32

PFCT TIC	MODES				
	(0,0)	(0,1)	(1,0)	(0,1)	(1,1)
1.00	3.5159	5.6546	22.0328	10.4810	20.0917
1.50	2.3438	4.9199	14.6910	11.3490	15.9075
2.00	1.7578	4.5653	11.0101	12.9300	14.3597
2.50	1.4062	4.3508	8.8103	14.4419	14.8167
3.00	1.1718	4.2071	7.3420	16.9461	13.9540
3.50	1.0043	4.1060	6.2931	19.3763	13.2612
4.00	0.8788	4.0319	5.5064	21.4587	12.9347
4.50	0.7811	3.9760	4.8945	23.3921	12.6729
5.00	0.7030	3.9314	4.4050	25.3702	12.4394
5.50	0.6392	3.8958	4.0045	27.3601	12.2363
6.00	0.5858	3.8692	3.6707	30.9752	12.1097
6.50	0.5407	3.8462	3.3833	33.4015	11.9900
7.00	0.5021	3.8309	3.1403	35.8464	11.8922
7.50	0.4686	3.8156	2.9365	38.1940	11.8270
8.00	0.4393	3.8027	2.7529	40.3466	11.7724
8.50	0.4135	3.7918	2.5910	43.2051	11.7153
9.00	0.3905	3.7824	2.4470	45.6683	11.6675
9.50	0.3699	3.7743	2.3182	48.1357	11.6264
0.00	0.3514	3.7673	2.2023	50.6064	11.5908

TABLE NO.= 1.01

RYX= 0.05000

PCISICNS RATIC= 0.20

NUMBER OF TERMS IN SERIES=30

ASPECT	MODES				
	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00	3.5159	5.6544	22.0328	10.4823	25.0360
1.50	2.3438	4.9199	14.6910	11.3505	18.8940
2.00	1.7578	4.5655	11.0101	12.9291	16.2128
2.50	1.4062	4.3509	8.8103	14.8393	14.7768
3.00	1.1718	4.2074	7.3420	16.9400	13.9013
3.50	1.0043	4.1058	6.2931	19.1686	13.3160
4.00	0.8788	4.0311	5.5064	21.4498	12.8986
4.50	0.7811	3.9743	4.8945	23.7825	12.5871
5.00	0.7030	3.9300	4.4050	26.1631	12.3470
5.50	0.6391	3.8947	4.0045	28.5521	12.1574
6.00	0.5858	3.8661	3.6707	30.9548	12.0047
6.50	0.5407	3.8426	3.3883	33.3977	11.8798
7.00	0.5021	3.8231	3.1463	35.8318	11.7764
7.50	0.4686	3.8066	2.9365	38.2773	11.6897
8.00	0.4393	3.7926	2.7529	40.7304	11.6163
8.50	0.4135	3.7805	2.5910	43.1893	11.5537
9.00	0.3905	3.7702	2.4470	45.6531	11.4998
9.50	0.3699	3.7611	2.3182	48.1209	11.4532
10.00	0.3514	3.7533	2.2032	50.5922	11.4125

TABLE NC.= 1.02

RYX= 0.06875

POISONS RATIO= 0.74

NUMBER OF TERMS IN SERIES=18

ASPECT	RATIO	MCDES				
		(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00		2.5157	5.9074	22.0319	11.3935	25.5046
1.50		2.3436	5.1836	16.6655	12.6470	19.4963
2.00		1.7575	4.8392	11.0066	14.6633	16.9190
2.50		1.4059	4.6284	8.8070	17.0078	15.5503
3.00		1.1715	4.4895	7.3393	19.5210	14.7120
3.50		1.0041	4.3937	6.2907	22.2097	14.1518
4.00		0.8786	4.3244	5.5042	24.9276	13.7515
4.50		0.7809	4.2725	4.8925	27.6961	13.4587
5.00		0.7028	4.2335	4.4031	30.4981	13.2232
5.50		0.6389	4.2032	4.0028	33.3242	13.0424
6.00		0.5856	4.1794	3.6691	36.1684	12.8976
6.50		0.5406	4.1602	3.3869	39.0263	12.7798
7.00		0.5019	4.1447	3.1449	41.8951	12.6828
7.50		0.4685	4.1319	2.9352	44.7725	12.6021
8.00		0.4392	4.1212	2.7517	47.6569	12.5342
8.50		0.4133	4.1108	2.5898	50.5471	12.4766
9.00		0.3904	4.1034	2.4459	53.4420	12.4273
9.50		0.3698	4.0971	2.3172	56.3410	12.3849
10.00		0.3513	4.0916	2.2013	59.2435	12.3481

TABLE NO.= 1.10

RY = C.06870

PCISIONS RATIO = 0.24

NUMBER OF TERMS IN SERIES = 24

ASPECT	MCDES				
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(2,1)
1.00	3.5157	5.9019	22.0219	11.8689	25.4893
2.00	2.3426	3.1770	16.6115	12.6115	19.4861
3.00	1.7575	4.8213	11.0065	14.6222	16.9009
4.00	1.4059	4.6055	8.8070	16.9645	15.5256
5.00	1.1715	4.4621	7.3392	19.4779	14.6881
6.00	1.0041	4.3617	6.2906	22.1658	14.1264
7.00	0.8785	4.2886	5.5041	24.8826	13.7328
8.00	0.7809	4.2334	4.8924	27.6955	13.4437
9.00	0.7028	4.1894	4.4030	30.4673	13.2052
10.00	0.6388	4.1556	4.0026	33.2931	13.0262
11.00	0.5856	4.1284	3.6690	36.1253	12.8827
12.00	0.5404	4.1063	3.3867	38.9975	12.7661
13.00	0.5019	4.0923	3.1447	41.8676	12.7012
14.00	0.4684	4.0776	2.9350	44.7464	12.6249
15.00	0.4392	4.0632	2.7515	47.6220	12.5613
16.00	0.4133	4.0493	2.5897	50.5234	12.5072
17.00	0.3903	4.0399	2.4458	53.4195	12.4614
18.00	0.3698	4.0319	2.3170	56.3195	12.4220
19.00	0.3513	4.0228	2.2011	59.2220	12.3880

TABLE NC. = 1.17

BYA = 0.05075

PCISICNS RATIC = 0.24

NUMBER OF TERMS IN SERIES = 20

ASPECT	MODES				
RATIC	(0,0)	(0,1)	(1,0)	(0,1)	(1,1)
1.00	3.5157	3.9016	22.0319	11.3686	23.4860
2.00	2.2486	3.1789	16.6106	12.6106	19.4698
3.00	1.7575	4.8315	1.0065	14.6198	16.8697
4.00	1.4059	4.6058	8.8070	16.9605	15.4780
5.00	1.1713	4.4622	7.3392	19.4726	14.6244
6.00	1.0041	4.3611	6.2904	22.1593	14.0529
7.00	0.8785	4.2870	5.5043	24.8752	13.6373
8.00	0.7808	4.2309	4.8924	27.6874	13.3281
9.00	0.7027	4.1873	4.4030	30.4573	13.0890
10.00	0.6388	4.1527	4.0026	33.1757	12.8996
11.00	0.5856	4.1247	3.6690	36.1434	12.7467
12.00	0.5405	4.1017	3.3867	38.9951	12.6215
13.00	0.5019	4.0826	3.1447	41.8639	12.5203
14.00	0.4689	4.0666	2.9350	44.7421	12.4469
15.00	0.4391	4.0530	2.7515	47.6276	12.3966
16.00	0.4133	4.0413	2.5896	50.5189	12.3669
17.00	0.3903	4.0313	2.4457	53.4150	12.3457
18.00	0.3698	4.0226	2.3170	56.3151	12.3415
19.00	0.3513	4.0150	2.2011	59.2186	12.3531

TABLE NO. = 1.12

RY = C.50000

PRECISIONS RATIO = 0.05

NUMBER OF TERMS IN SERIES = 18

ASPECT	MOSES				
RATIO	(0,0)	(0,1)	(1,0)	(0,1)	(1,1)
1.00	9.5158	8.4067	11.8164	21.6875	20.6624
1.50	2.3436	7.7078	14.6862	27.7178	25.8641
2.00	1.7578	7.3548	11.0146	34.6349	22.8149
2.50	1.4062	7.1629	8.8116	41.9401	22.6993
3.00	1.2718	7.0535	7.3429	49.4485	22.0067
3.50	1.0044	6.9750	6.2938	57.0733	21.5440
4.00	0.8788	6.9175	5.5071	64.7697	21.2196
4.50	0.7812	6.8805	4.8951	72.5138	20.9836
5.00	0.7030	6.8533	4.4056	80.3912	20.8071
5.50	0.6391	6.8328	4.0051	88.0929	20.6718
6.00	0.5859	6.8170	3.6713	95.9126	20.5659
6.50	0.5408	6.8258	3.3889	103.7464	20.4816
7.00	0.5022	6.7945	3.1468	111.5910	20.4135
7.50	0.4687	6.7864	2.9370	119.4444	20.4384
8.00	0.4394	6.7797	2.7834	127.3049	20.3965
8.50	0.4135	6.7741	2.5915	135.1712	20.3616
9.00	0.3905	6.7694	2.4475	143.0424	20.3322
9.50	0.3700	6.7655	2.3187	150.9178	20.3072
10.00	0.3515	6.7620	2.2027	158.7966	20.2858

TABLE NC. = 1.20

RYX= 0.50000

POISONS RATIO= 0.05

NUMBER OF TERMS IN SERIES=24

ASPECT		MCDES				
RATIO		(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00		3.5158	8.3825	21.7346	21.6539	30.6397
1.50		2.3437	7.6688	14.6897	27.6244	25.8226
2.00		1.7580	7.3139	11.0173	34.5432	23.7797
2.50		1.4060	7.1099	8.8138	41.8607	22.6637
3.00		1.1720	6.9788	7.3449	49.3787	21.9767
3.50		1.0046	6.8966	6.2956	57.0105	21.5179
4.00		0.8790	6.8367	5.5086	64.7130	21.1967
4.50		0.7813	6.7933	4.8966	72.4572	21.0210
5.00		0.7032	6.7608	4.4069	80.2583	20.8536
5.50		0.6393	6.7360	4.0063	88.0496	20.7297
6.00		0.5860	6.7165	3.6724	95.8727	20.6318
6.50		0.5409	6.6898	3.3899	103.7092	20.5544
7.00		0.5023	6.6767	3.1478	111.5564	20.4921
7.50		0.4688	6.6623	2.9374	119.4120	20.4413
8.00		0.4395	6.6570	2.7543	127.2745	20.3994
8.50		0.4136	6.6466	2.5923	135.1395	20.3644
9.00		0.3907	6.6432	2.4483	143.0118	20.2999
9.50		0.3701	6.6515	2.3194	150.8921	20.2738
10.00		0.3516	6.6471	2.2035	158.7722	20.2515

TABLE NO.= 1.21

RYX= 0.50000

POSITIONS RATIO= 0.05

NUMBER OF TERMS IN SERIES=30

ASPECT		MCDES				
RATIO		(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00		3.5158	8.3827	21.6272	21.6272	30.6032
1.50		2.3437	7.6659	14.6861	27.6380	25.7427
2.00		1.7577	7.3136	11.0146	34.5551	23.6550
2.50		1.4061	7.1070	8.8115	41.8577	22.5128
3.00		1.1718	6.9754	7.3428	49.2779	21.8022
3.50		1.0043	6.8859	6.2938	56.8865	21.3256
4.00		0.8788	6.8222	5.5070	64.7192	20.9892
4.50		0.7811	6.7751	4.8950	72.4604	20.7430
5.00		0.7030	6.7393	4.4055	80.2416	20.5573
5.50		0.6391	6.7115	4.0050	88.0598	20.4141
6.00		0.5858	6.6894	3.6712	95.8813	20.3012
6.50		0.5408	6.6727	3.3888	103.7175	20.2109
7.00		0.5021	6.6572	3.1467	111.5639	20.1375
7.50		0.4687	6.6453	2.9269	119.4189	20.0771
8.00		0.4394	6.6353	2.7533	127.2808	20.0268
8.50		0.4135	6.6269	2.5914	135.1484	19.9845
9.00		0.3905	6.6198	2.4474	143.0208	19.9485
9.50		0.3700	6.6137	2.3186	150.8972	19.9178
10.00		0.3515	6.6084	2.2027	158.7769	19.8914

TABLE NC.= 1.22

RYK= 1.00000

POISONS RATIO= 0.20

NUMBER OF TERMS IN SERIES=18

ASPECT	MODES				
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
1.00	3.4937	8.5441	21.4400	27.4562	31.1718
1.50	2.3212	7.8173	14.4184	36.1142	26.4598
2.00	1.7363	7.4607	10.8050	47.2384	24.3758
2.50	1.3864	7.2617	8.6350	57.8410	23.1752
3.00	1.1537	7.1403	7.1896	68.6772	22.4324
3.50	0.9879	7.0619	6.1582	79.6225	21.9330
4.00	0.8628	7.0085	5.3854	90.6509	21.6182
4.50	0.7673	6.9706	4.7848	101.7037	21.3725
5.00	0.6903	6.9429	4.3028	112.7843	21.1903
5.50	0.6273	6.9219	3.9123	123.8848	21.0517
6.00	0.5748	6.9058	3.5854	134.9999	20.9442
6.50	0.5305	6.8931	3.3089	146.1262	20.8337
7.00	0.4925	6.8830	3.0720	157.2613	20.7632
7.50	0.4596	6.8747	2.8668	168.4034	20.7056
8.00	0.4308	6.8679	2.6873	179.5569	20.6580
8.50	0.4054	6.8623	2.5290	190.7095	20.6182
9.00	0.3829	6.8575	2.3882	201.8660	20.5845
9.50	0.3627	6.8535	2.2624	213.0025	20.5559
10.00	0.3445	6.8500	2.1491	224.1881	20.5314

TABLE NO.= 1.30

RYL= 1.00000

PRECISIONS RATIO= 0.80

NUMBER OF TERMS IN SERIES=24

ASPECT						
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)	
1.00	3.4926	8.5275	21.4345	27.3854	31.1496	
1.50	2.3196	7.7892	14.4141	36.0083	26.4243	
2.00	1.7345	7.4204	10.7992	46.9760	24.3117	
2.50	1.3844	7.2090	8.6281	56.0593	23.1381	
3.00	1.1517	7.0767	7.1819	68.7330	22.4022	
3.50	0.9858	6.9886	6.1302	79.6517	21.9446	
4.00	0.8617	6.9189	5.3774	90.6529	21.6091	
4.50	0.7654	6.8727	4.7770	101.7011	21.3671	
5.00	0.6884	6.8383	4.2972	112.7796	21.1873	
5.50	0.6255	6.8118	3.9050	123.8792	21.0270	
6.00	0.5731	6.7910	3.5784	134.9943	20.9180	
6.50	0.5289	6.7766	3.3022	146.1209	20.8314	
7.00	0.4910	6.7633	3.0656	157.2565	20.7616	
7.50	0.4581	6.7500	2.8607	168.3992	20.7045	
8.00	0.4293	6.7409	2.6814	179.5475	20.6573	
8.50	0.4044	6.7332	2.5234	190.7005	20.6178	
9.00	0.3816	6.7268	2.3829	201.8573	20.5845	
9.50	0.3615	6.7212	2.2574	213.0173	20.5567	
10.00	0.3434	6.7165	2.1443	224.1775	20.5317	

TABLE NO.= 1.31

RYA= 1.00000

POISSONS RATIO= 0.50

NUMBER OF TERMS IN SERIES=30

ASPECT	MCDPS				
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
0.00	3.4919	8.5268	21.4308	27.3504	31.3131
1.50	2.3185	7.7880	14.4134	35.9476	26.2340
2.00	1.7332	7.4181	10.7985	46.8786	24.1940
2.50	1.3830	7.2041	8.6273	57.8879	22.9126
3.00	1.1503	7.0682	7.1809	69.4690	22.2276
3.50	0.9845	6.9762	6.1490	79.7350	21.7137
4.00	0.8604	6.9110	5.3760	90.6840	21.3503
4.50	0.7641	6.8628	4.7754	101.7141	21.0840
5.00	0.6872	6.8264	4.2955	112.7841	20.8834
5.50	0.6244	6.7981	3.9031	123.8791	20.7286
6.00	0.5720	6.7758	3.5765	134.9914	20.6068
6.50	0.5278	6.7579	3.3003	146.1164	20.5091
7.00	0.4900	6.7432	3.0637	157.2510	20.4203
7.50	0.4512	6.7312	2.8588	168.3930	20.3432
8.00	0.4285	6.7211	2.6795	179.5410	20.2711
8.50	0.4032	6.7127	2.5215	190.6938	20.2056
9.00	0.3807	6.7055	2.3810	201.8506	20.1470
9.50	0.3606	6.6993	2.2554	213.0106	20.0941
10.00	0.3426	6.6940	2.1424	224.1734	20.0467

TABLE NO.= 1.32

RYX= 0.05000

POISONS RATIO= 0.20

NUMBER OF TERMS IN SERIES=18

ASPECT	* *	MODES					* *
RATIO	* *	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)	* *
*****							
0.10	*	35.1600	35.4374	220.3457	36.2938	220.6619	*
0.20	*	17.5800	18.1302	110.1728	19.7713	110.8042	*
0.30	*	11.7200	12.5334	73.4485	14.8458	74.3934	*
0.40	*	8.7900	9.8528	55.0865	12.7150	56.3422	*
0.50	*	7.0319	8.3265	44.0693	11.6313	45.6323	*
0.60	*	5.8599	7.3658	36.7249	11.0359	*****	*
0.70	*	5.0228	6.7186	31.4798	10.7049	33.6253	*
0.80	*	4.3949	6.2596	27.4056	10.5366	29.9888	*
0.90	*	3.9065	5.9197	24.4797	10.4794	27.2147	*
1.00	*	3.5159	5.6592	22.0328	10.5042	25.0432	*
2.00	*	1.7578	4.5812	11.0101	12.9688	16.2533	*
3.00	*	1.1718	4.2313	7.3420	16.9854	13.9809	*
4.00	*	0.8788	4.0581	5.5064	21.4950	12.9957	*
5.00	*	0.7030	3.9661	4.4051	26.1984	12.4575	*
6.00	*	0.5859	3.9141	3.6708	31.0026	12.1247	*
7.00	*	0.5021	3.8783	3.1464	35.8638	11.9041	*
8.00	*	0.4393	3.8513	2.7530	40.7606	11.7506	*
9.00	*	0.3905	3.8343	2.4471	45.6811	11.6398	*
10.00	*	0.3515	3.8219	2.2024	50.6180	11.5574	*

TABLE NO.= 2.00

RYA = 0.0073

POISONS RATIO = 0.14

NUMBER OF TERMS IN SERIES = 18

ASPECT	MODES				
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(2,0)
0.10	30.1600	35.4781	210.3457	36.4589	320.7311
0.20	17.5800	18.2102	110.1728	20.0823	110.9033
0.30	11.7199	12.6505	73.4488	15.2737	74.5415
0.40	8.7899	10.0030	55.0865	13.3319	56.5358
0.50	7.0318	8.3048	44.0698	12.2171	45.8806
0.60	5.8598	7.5640	36.7264	11.6808	38.3731
0.70	5.0226	6.9374	28.7047	11.4066	33.9563
0.80	5.3947	6.4914	27.5332	11.2970	30.3587
0.90	3.9064	6.1614	24.4786	11.3022	27.6281
1.00	3.5157	5.9074	22.0319	11.3935	25.5046
2.00	1.7575	4.8392	11.0066	14.6633	16.9190
3.00	1.1715	4.4895	7.3393	19.5210	14.7120
4.00	0.8185	4.3244	5.5042	24.9276	13.7515
5.00	0.7023	4.2735	4.4031	30.4961	13.2232
6.00	0.5856	4.1794	3.6691	36.684	12.8976
7.00	0.5019	4.1447	3.1449	41.8951	12.6828
8.00	0.4592	4.1212	2.7517	47.6569	12.5142
9.00	0.3904	4.1034	2.4459	53.4420	12.4273
10.00	0.3513	4.0916	2.2013	59.2435	12.3481

TABLE NO. = 2.10

RYA= 0.50000

POISONS RATIO= 0.05

NUMBER OF TERMS IN SERIES=18

ASPECT		MCDES				
RATIO		(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
0.10		35.1600	36.0417	220.3457	38.6922	221.3511
0.20		17.5800	19.2944	110.1729	24.0097	112.1745
0.30		11.7199	14.1741	73.4483	20.3137	76.4315
0.40		8.7899	11.8663	55.0888	19.0429	58.9990
0.50		7.0319	10.6112	44.0653	18.6885	48.9050
0.60		5.8598	9.8375	36.7228	18.8211	42.4422
0.70		5.0227	9.3130	31.4774	19.2683	37.4622
0.80		4.3948	8.9319	27.5438	19.9388	34.7916
0.90		3.9060	8.6398	*****	20.7742	32.4521
1.00		3.5158	8.4067	21.8164	21.6875	30.6624
2.00		1.7578	7.3548	11.0146	34.6349	23.8149
3.00		1.1718	7.0535	7.3429	49.4485	22.0067
4.00		0.8788	6.9175	5.5029	64.7697	21.2196
5.00		0.7030	6.8533	4.4056	80.2912	20.8071
6.00		0.5859	6.8170	3.6713	95.9126	20.5659
7.00		0.5022	6.7945	3.1468	111.5910	20.4135
8.00		0.4394	6.7797	2.7534	127.3049	20.3965
9.00		0.3906	6.7694	2.4475	143.0424	20.3322
10.00		0.3515	6.7581	2.2027	158.7966	20.2858

TABLE NO.= 2.20

RYX= 1.00000

POISONS RATIO= 0.30

NUMBER OF TERMS IN SERIES=18

ASPECT	MCDES				
RATIO	(0,0)	(0,1)	(1,0)	(0,2)	(1,1)
0.10	35.1577	36.0155	220.3380	38.6872	221.4828
0.20	17.5741	19.2930	110.1721	24.3458	112.4926
0.30	11.7104	14.2261	73.5104	21.0952	*****
0.40	8.7771	11.9669	54.1262	20.3201	59.3435
0.50	7.0164	10.7433	43.9152	20.5153	49.6916
0.60	5.8423	9.9842	36.6652	21.2297	40.9240
0.70	5.0031	9.4639	31.5357	22.2406	38.0155
0.80	4.3741	9.0805	27.9258	23.2774	35.1180
0.90	3.8849	8.7831	23.2218	26.4118	32.8921
1.00	3.4937	8.5441	21.4400	27.4561	31.1718
2.00	1.7363	7.4607	10.8050	47.2348	24.3758
3.00	1.1537	7.1403	7.1896	68.6772	22.4324
4.00	0.8638	7.0035	5.3854	90.6509	21.6183
5.00	0.6903	6.9429	4.3048	112.7843	21.1906
6.00	0.5748	6.9058	3.5854	134.9999	20.9442
7.00	0.4925	6.8830	3.0720	157.2613	20.7623
8.00	0.4303	6.8679	2.6873	179.5569	20.6580
9.00	0.3829	6.8575	2.3882	201.8660	20.5845
10.00	0.3445	6.8500	2.1491	224.1881	20.5314

TABLE NO.= 2.30

RYX= 0.06875

MODE (0,0)

## EFFECT OF POISONS RATIO ON FREQUENCY PARAMETER.

ASPECT	POISONS RATIO				
RATIO	0.025	0.05	0.10	0.20	0.30
1.00	3.5160	3.5160	3.5159	3.5158	3.5155
1.50	2.3440	2.3440	2.3439	2.3437	2.3433
2.00	1.7580	1.7580	1.7579	1.7577	1.7573
2.50	1.4064	1.4064	1.4063	1.4061	1.4057
3.00	1.1720	1.1720	1.1719	1.1717	1.1713
3.50	1.0046	1.0046	1.0045	1.0043	1.0039
4.00	0.8790	0.8790	0.8789	0.8787	0.8783
4.50	0.7813	0.7813	0.7813	0.7810	0.7807
5.00	0.7032	0.7032	0.7031	0.7029	0.7026
5.50	0.6393	0.6393	0.6392	0.6390	0.6387
6.00	0.5860	0.5860	0.5859	0.5857	0.5854
6.50	0.5409	0.5409	0.5409	0.5407	0.5404
7.00	0.5023	0.5023	0.5022	0.5020	0.5017
7.50	0.4688	0.4688	0.4687	0.4686	0.4683
8.00	0.4395	0.4395	0.4394	0.4393	0.4390
8.50	0.4136	0.4136	0.4136	0.4134	0.4132
9.00	0.3907	0.3907	0.3906	0.3905	0.3902
9.50	0.3701	0.3701	0.3701	0.3699	0.3697
10.00	0.3516	0.3516	0.3516	0.3514	0.3512

TABLE NO.= 3.00

RYK= 0.06875

MODE (0,1)

## EFFECT OF POISONS RATIO ON FREQUENCY PARAMETER.

ASPECT	POISONS RATIO				
RATIO	0.025	0.05	0.10	0.20	0.30
1.00	6.0017	5.9904	5.9676	5.9262	5.8789
1.50	5.2852	5.2736	5.2502	5.2085	5.1545
2.00	4.9366	4.9248	4.9010	4.8590	4.8037
2.50	4.7358	4.7235	4.6989	4.6490	4.5980
3.00	4.5912	4.5789	4.5541	4.5100	4.4531
3.50	4.4971	4.4846	4.4594	4.4139	4.3566
4.00	4.4299	4.4171	4.3916	4.3398	4.2873
4.50	4.3852	4.3723	4.3463	4.2892	4.2361
5.00	4.3472	4.3348	4.3079	4.2549	4.1973
5.50	4.3178	4.3046	4.2782	4.2253	4.1707
6.00	4.2913	4.2782	4.2548	4.2011	4.1466
6.50	4.2732	4.2599	4.2333	4.1820	4.1248
7.00	4.2585	4.2452	4.2184	4.1666	4.1116
7.50	4.2464	4.2330	4.2061	4.1518	4.0991
8.00	4.2363	4.2229	4.1960	4.1415	4.0883
8.50	4.2279	4.2145	4.1889	4.1328	4.0789
9.00	4.2208	4.2073	4.1815	4.1270	4.0713
9.50	4.2147	4.2012	4.1740	4.1192	4.0647
10.00	4.2094	4.1959	4.1687	4.1138	4.0591

TABLE NO.= 3.10

RYX = 0.07

MODE(1,0)

EFFECT OF PRECISIONS RATIO ON FREQUENCY PARAMETER.

ASPECT						
			RYX			
RATIO		0.05	0.25	0.50	0.75	1.00
1.00		22.0345	22.0345	22.0341	22.0327	22.0303
1.50		12.8075	12.7832	12.7718	12.6895	*****
2.00		11.0172	11.0168	11.0155	11.0099	11.0005
2.50		8.8138	8.8135	8.8127	8.8091	8.8032
3.00		7.3448	7.3446	7.3439	7.3410	7.3361
3.50		6.2955	6.2954	6.2947	6.2922	6.2879
4.00		5.5086	5.5085	5.5079	5.5056	5.5017
4.50		4.8965	4.8964	4.8959	4.8937	4.8902
5.00		4.4069	4.4068	4.4063	4.4043	4.4010
5.50		4.0062	4.0061	4.0057	4.0039	4.0008
6.00		3.6724	3.6723	3.6719	3.6701	3.6673
6.50		3.3899	3.3898	3.3894	3.3878	3.3851
7.00		3.1478	3.1477	3.1473	3.1458	3.1432
7.50		2.9374	2.9373	2.9369	2.9360	2.9336
8.00		2.7543	2.7542	2.7539	2.7525	2.7502
8.50		2.5923	2.5922	2.5919	2.5906	2.5884
9.00		2.4483	2.4482	2.4479	2.4466	2.4446
9.50		2.3194	2.3193	2.3190	2.3179	2.3159
10.00		2.2034	2.2034	2.2031	2.2019	2.2001

TABLE NO. = 3.20

RYK= 0.06875

MODE (1,1)

EFFECT OF POISONS RATIO ON FREQUENCY PARAMETER.

ASPECT	POISONS RATIO				
RATIO	0.025	0.05	0.10	0.20	0.30
1.00	25.6270	25.6125	25.5785	25.5186	25.4609
1.50	19.6998	19.6771	19.6309	19.5356	19.4361
2.00	17.1463	17.1206	17.0687	16.9624	16.8528
2.50	15.7972	15.7692	15.7126	15.5973	15.4788
3.00	14.9807	14.9509	14.8909	14.7685	14.6365
3.50	14.4251	14.3940	14.3313	14.2037	14.0731
4.00	14.0348	14.0025	13.9375	13.8052	13.6701
4.50	13.7446	13.7114	13.6443	13.5081	13.3692
5.00	13.5223	13.4881	13.4193	13.2798	13.1439
5.50	13.3478	13.3129	13.2426	13.1001	12.9550
6.00	13.2085	13.1729	13.1014	12.9563	12.8087
6.50	13.0954	13.0593	12.9866	12.8359	12.6896
7.00	13.0026	12.9660	12.8923	12.7432	12.5915
7.50	12.9253	12.8883	12.8183	12.6631	12.5098
8.00	12.8605	12.8231	12.7479	12.5957	12.4411
8.50	12.8056	12.7679	12.6921	12.5386	12.3827
9.00	12.7587	12.7207	12.6443	12.4898	12.3328
9.50	12.7183	12.6801	12.6032	12.4477	12.2898
10.00	12.6834	12.6449	12.5676	12.4113	12.2525

TABLE NO.= 3.30

POI SIONS RATIO= 0.25

MODE(0,0)

EFFECT OF RYX ON FREQUENCY PARAMETER.

ASPECT	RYX				
RATIO	0.05	0.25	0.50	0.75	1.00
1.00	3.5158	3.5138	3.5102	3.5058	3.5009
1.50	2.3437	2.3415	2.3377	2.3334	2.3286
2.00	1.7577	1.7554	1.7518	1.7477	1.7433
2.50	1.4061	1.4093	1.4005	1.3968	1.3928
3.00	1.1717	1.1696	1.1665	1.1631	1.1595
3.50	1.0042	1.0023	0.9994	0.9964	0.9932
4.00	0.8787	0.8768	0.8742	0.8715	0.8686
4.50	0.7810	0.7793	0.7769	0.7744	0.7718
5.00	0.7029	0.7013	0.6991	0.6967	0.6944
5.50	0.6390	0.6375	0.6354	0.6333	0.6311
6.00	0.5857	0.5843	0.5824	0.5804	0.5784
6.50	0.5407	0.5393	0.5375	0.5357	0.5338
7.00	0.5020	0.5008	0.4991	0.4974	0.4956
7.50	0.4685	0.4674	0.4658	0.4642	0.4625
8.00	0.4393	0.4381	0.4366	0.4351	0.4335
8.50	0.4134	0.4123	0.4109	0.4095	0.4080
9.00	0.3904	0.3894	0.3881	0.3867	0.3853
9.50	0.3699	0.3689	0.3676	0.3663	0.3650
10.00	0.3514	0.3505	0.3492	0.3480	0.3467

TABLE NO.= 4.00

POISSONS RATIO= 0.25

MODE(0,1)

EFFECT OF RYX ON FREQUENCY PARAMETER.

ASPECT RATIO	RYX				
	0.05	0.25	0.50	0.75	1.00
1.00	5.6409	7.1700	7.9534	8.4276	8.7582
1.50	4.9061	6.4601	7.2479	7.7159	8.0362
2.00	4.5616	6.1120	6.8874	7.3569	7.6842
2.50	4.3511	5.8979	6.6850	7.1583	7.4886
3.00	4.2118	5.7668	6.5600	7.0372	7.3702
3.50	4.1136	5.6843	6.4781	6.9586	7.2938
4.00	4.0427	5.6227	6.4219	6.9049	7.2418
4.50	3.9898	5.5745	6.3818	6.8668	7.2050
5.00	3.9494	5.5455	6.3522	6.8388	7.1781
5.50	3.9184	5.5197	6.3299	6.8177	7.1578
6.00	3.8929	5.4979	6.3143	6.8015	7.1422
6.50	3.8729	5.4826	6.3003	6.7887	7.1299
7.00	3.8569	5.4704	6.2890	6.7784	7.1201
7.50	3.8430	5.4603	6.2799	6.7701	7.1121
8.00	3.8317	5.4521	6.2724	6.7632	7.1055
8.50	3.8206	5.4452	6.2661	6.7575	7.1001
9.00	3.8127	5.4393	6.2608	6.7527	7.0959
9.50	3.8075	5.4344	6.2562	6.7486	7.0916
10.00	3.8002	5.4301	6.2524	6.7451	7.0882

POISONS RATIO= 0.25

MODE(1,0)

EFFECT OF RYX ON FREQUENCY PARAMETER.

ASPECT				RYX		
RATIO		0.05	0.25	0.50	0.75	1.00
1.00		22.0318	22.0473	21.7218	21.6005	21.6327
1.50		14.6917	14.6355	14.5965	14.5516	14.5042
2.00		11.0057	10.9837	10.9483	10.9108	10.8718
2.50		8.8083	8.7863	8.7560	8.7241	8.6912
3.00		7.3405	7.3210	7.2943	7.2668	7.2383
3.50		6.2918	6.2743	6.2508	6.2263	6.2012
4.00		5.5052	5.4894	5.4683	5.4463	5.4239
4.50		4.8934	4.8790	4.8598	4.8400	4.8197
5.00		4.4040	4.3907	4.3732	4.3551	4.3367
5.50		4.0036	3.9913	3.9751	3.9585	3.9416
6.00		3.6699	3.6585	3.6435	3.6281	3.6125
6.50		3.3875	3.3769	3.3629	3.3486	3.3342
7.00		3.1455	3.1355	3.1225	3.1091	3.0956
7.50		2.9358	2.9264	2.9141	2.9016	2.8890
8.00		2.7523	2.7434	2.7318	2.7201	2.7082
8.50		2.5904	2.5819	2.5710	2.5599	2.5487
9.00		2.4464	2.4384	2.4281	2.4176	2.4069
9.50		2.3177	2.3100	2.3002	2.2902	2.2801
10.00		2.2018	2.1945	2.1851	2.1756	2.1660

TABLE NO. = 4.20

POISSONS RATIO= 0.25

MODE(1,1)

EFFECT OF RYX ON FREQUENCY PARAMETER.

ASPECT	RYX				
RATIO	0.05	0.25	0.50	0.75	1.00
1.00	25.0205	27.9045	29.7041	30.8570	39.6121
1.50	18.8785	22.6490	24.7732	26.0949	27.0325
2.00	16.2122	20.3981	22.6467	24.0190	24.9574
2.50	14.7928	19.1838	21.4526	22.8338	23.7998
3.00	13.9292	18.4092	20.7101	22.1012	23.0730
3.50	13.3627	17.8854	20.2261	21.6386	22.5859
4.00	12.9442	17.5152	19.8529	21.2951	22.2847
4.50	12.6386	17.2415	19.5925	21.0515	22.0310
5.00	12.4031	17.0600	19.3963	20.8707	21.8523
5.50	12.2174	16.8981	19.2453	20.7332	21.7155
6.00	12.0681	16.7820	19.1266	20.6264	21.6339
6.50	11.9463	16.6857	19.0319	20.5420	21.5521
7.00	11.8458	16.6072	18.9552	20.4741	21.4866
7.50	11.7618	16.5429	18.8922	20.4189	21.4332
8.00	11.6910	16.4896	18.9160	20.3732	21.3892
8.50	11.6308	16.4450	18.8758	20.3352	21.3525
9.00	11.5792	16.4073	18.8419	20.3031	21.3216
9.50	11.5347	16.3752	18.8130	20.2759	21.2953
10.00	11.4960	16.3477	18.7883	20.2525	21.2728

TABLE NO.= 4.30

PYX = 0.06875

POISONS RATIO = 0.24

NUMBER OF TERMS IN SERIES = 18  
FREQUENCY PARAMETER FOR FREE PLATE

ASPECT RATIO	MODES				
	(0,0)	(0,1)	(1,0)	(1,1)	(2,2)
0.10	-0.0000	-0.0000	-0.0000	7.6518	0.5857
0.20	-0.0000	-0.0000	-0.0000	7.6604	1.1717
0.30	-0.0000	-0.0000	-0.0000	7.6738	1.7572
0.40	-0.0000	-0.0000	-0.0000	7.6910	2.3430
0.50	-0.0000	-0.0000	-0.0000	7.7109	2.9288
0.60	-0.0000	-0.0000	-0.0000	7.7322	3.5148
0.70	-0.0000	-0.0000	-0.0000	7.7538	4.1007
0.80	-0.0000	-0.0000	-0.0000	7.7750	4.6866
0.90	-0.0000	-0.0000	-0.0000	7.7951	5.2724
1.00	-0.0000	-0.0000	-0.0000	7.8134	5.8582
2.00	-0.0000	-0.0000	-0.0000	7.8873	11.8232
3.00	-0.0000	-0.0000	-0.0000	7.8527	17.5981
4.00	-0.0000	-0.0000	-0.0000	7.8051	23.4555
5.00	-0.0000	-0.0000	-0.0000	7.7673	29.3431
6.00	-0.0000	-0.0000	-0.0000	7.7399	35.2011
7.00	-0.0000	-0.0000	-0.0000	7.7203	41.0658
8.00	-0.0000	-0.0000	-0.0000	7.7061	46.9315
9.00	-0.0000	-0.0000	-0.0000	7.6955	52.7975
10.00	-0.0000	-0.0000	-0.0000	7.6876	58.6636

TABLE NO. = 5.00

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 1.0

RYX	*	POISONS RATIO					*
	*						*
	*	0.025	0.05	0.10	0.20	0.30	*
	*						*
0.05	*	3.5160	3.5160	3.5160	3.5159	3.5159	*
	*						*
0.25	*	3.5160	3.5159	3.5157	3.5146	3.5127	*
	*						*
0.50	*	3.5159	3.5158	3.5151	3.5125	3.5074	*
	*						*
0.75	*	3.5159	3.5156	3.5145	3.5096	3.5010	*
	*						*
1.00	*	3.5159	3.5154	3.5138	3.5066	3.4937	*
	*						*

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 2.0

RYX	*	POISONS RATIO					*
	*						*
	*	0.025	0.05	0.10	0.20	0.30	*
	*						*
0.05	*	1.7580	1.7580	1.7579	1.7578	1.7575	*
	*						*
0.25	*	1.7580	1.7579	1.7576	1.7564	1.7542	*
	*						*
0.50	*	1.7579	1.7578	1.7571	1.7541	1.7490	*
	*						*
0.75	*	1.7579	1.7576	1.7564	1.7516	1.7429	*
	*						*
1.00	*	1.7579	1.7575	1.7558	1.7488	1.7363	*
	*						*

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 3.0

RYX	*	POISICNS RATIO					*
	*	0.025	0.05	0.10	0.20	0.30	*
	*						*
0.05	*	1.1720	1.1720	1.1719	1.1718	1.1715	*
0.25	*	1.1720	1.1719	1.1716	1.1705	1.1685	*
0.50	*	1.1719	1.1718	1.1711	1.1685	1.1639	*
0.75	*	1.1719	1.1717	1.1706	1.1664	1.1589	*
1.00	*	1.1719	1.1715	1.1701	1.1642	1.1536	*

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 4.0

*	*						*	
*	RYX	*	POISICNS RATIO				*	
*	*	*					*	
*	*	*	0.025	0.05	0.10	0.20	0.30	*
*	*	*						*
*	*	*	-----					*
*	*	*						*
*	0.05	*	0.8790	0.8790	0.8789	0.8788	0.8785	*
*	*	*						*
*	0.25	*	0.8790	0.8789	0.8787	0.8776	0.8759	*
*	*	*						*
*	0.50	*	0.8790	0.8788	0.8783	0.8760	0.8721	*
*	*	*						*
*	0.75	*	0.8789	0.8787	0.8778	0.8742	0.8679	*
*	*	*						*
*	1.00	*	0.8789	0.8786	0.8774	0.8724	0.8636	*
*	*	*						*

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 5.0

RYX	POISSONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	0.7032	0.7032	0.7032	0.7030	0.7028	
0.25	0.7032	0.7031	0.7029	0.7020	0.7004	
0.50	0.7032	0.7030	0.7026	0.7006	0.6972	
0.75	0.7031	0.7030	0.7022	0.6991	0.6937	
1.00	0.7031	0.7029	0.7018	0.6976	0.6902	

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 6.0

RYX	POISSONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	0.5860	0.5860	0.5860	0.5858	0.5856	
0.25	0.5860	0.5859	0.5850	0.5849	0.5835	
0.50	0.5860	0.5859	0.5854	0.5837	0.5807	
0.75	0.5859	0.5858	0.5851	0.5824	0.5778	
1.00	0.5859	0.5857	0.5848	0.5812	0.5748	

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 7.0

RYX	*	POISSONS RATIO					*
		0.025	0.05	0.10	0.20	0.30	
0.05	*	0.5023	0.5023	0.5022	0.5021	0.5019	*
0.25	*	0.5023	0.5022	0.5020	0.5013	0.5001	*
0.50	*	0.5023	0.5022	0.5018	0.5003	0.4976	*
0.75	*	0.5022	0.5021	0.5015	0.4992	0.4951	*
1.00	*	0.5022	0.5020	0.5012	0.4980	0.4924	*

DESIGN TABLES FOR MODE(0,0) OF CANTILEVER PLATE  
OF ASPECT RATIO= 8.0

RYX	*	POISSONS RATIO					*
		0.025	0.05	0.10	0.20	0.30	
0.05	*	0.4395	0.4395	0.4395	0.4393	0.4392	*
0.25	*	0.4395	0.4394	0.4393	0.4386	0.4375	*
0.50	*	0.4395	0.4394	0.4390	0.4377	0.4353	*
0.75	*	0.4395	0.4393	0.4388	0.4367	0.4331	*
1.00	*	0.4394	0.4393	0.4386	0.4357	0.4308	*

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 1.0

RYX	POISONS RATIO				
	0.025	0.05	0.10	0.20	0.30
0.05	5.7175	5.7085	5.6955	5.6592	5.6226
0.25	7.4505	7.4198	7.3655	7.2363	7.0980
0.50	8.4485	8.4067	8.2897	8.0685	7.8352
0.75	9.1136	9.0541	8.8956	8.5888	8.2609
1.00	9.6250	9.5477	9.3517	8.9637	8.5441

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 2.0

RYX	*	POISONS RATIO					*
		0.025	0.05	0.10	0.20	0.30	
0.05	*	4.6405	4.6313	4.6127	4.5812	4.5368	*
0.25	*	6.4152	6.3825	6.3164	6.1812	6.0416	*
0.50	*	7.4105	7.3548	7.2508	7.0154	6.7636	*
0.75	*	8.0833	8.0067	7.8506	7.5261	7.1824	*
1.00	*	8.6029	8.5068	8.3107	7.9000	7.4607	*

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 3.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	4.3000	4.2903	4.2707	4.2313	4.1913	
0.25	6.0797	6.0459	5.9776	5.8381	5.6997	
0.50	7.1108	7.0535	6.9279	6.6910	6.4320	
0.75	7.7965	7.7167	7.5544	7.2121	6.8574	
1.00	8.3259	8.2272	8.0226	7.5925	7.1403	

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 4.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	4.1282	4.1183	4.0983	4.0581	4.0221	
0.25	5.9443	5.9095	5.8393	5.6915	5.5443	
0.50	6.9768	6.9175	6.8014	6.5496	6.2914	
0.75	7.6710	7.5899	7.4249	7.0827	6.7220	
1.00	8.2063	8.1051	7.8986	7.4675	7.0085	

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 5.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	4.0430	4.0327	4.0116	3.9661	3.9284	
0.25	5.8675	5.8323	5.7644	5.6160	5.4668	
0.50	6.9133	6.8533	6.7344	6.4839	6.2203	
0.75	7.6145	7.5303	7.3660	7.0206	6.6543	
1.00	8.1508	8.0487	7.8424	7.4057	6.9429	

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 6.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	3.9872	3.9768	3.9560	3.9141	3.8717	
0.25	5.8263	5.9708	5.7190	5.5726	5.4246	
0.50	6.8790	6.8170	6.6946	6.4425	6.1798	
0.75	7.5791	7.4968	7.3306	6.9834	6.6159	
1.00	8.1206	8.0170	7.8081	7.3709	6.9058	

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 9.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	3.9090	3.8984	3.8772	3.8343	3.7924	
0.25	5.7723	5.7358	5.6632	5.5155	5.3626	
0.50	6.8304	6.7694	6.6459	6.3916	6.1266	
0.75	7.5361	7.4532	7.2844	6.9345	6.5657	
1.00	8.0793	7.9760	7.7651	7.3255	6.8757	

DESIGN TABLES FOR MODE(0,1) OF CANTILEVER PLATE  
OF ASPECT RATIO= 10.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	3.8968	3.8852	3.8660	3.8219	3.7724	
0.25	5.7632	5.7272	5.6544	5.5062	5.3532	
0.50	6.8230	6.7620	6.6383	6.3836	6.1182	
0.75	7.5294	7.4454	7.2775	6.9271	6.5579	
1.00	8.0730	7.9697	7.7583	7.3183	6.8500	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 1.0

RYX	POISONS RATIO				
	0.025	0.05	0.10	0.20	0.30
0.05	7.4741	7.4539	7.4133	7.3313	7.2483
0.25	11.3354	11.2558	11.1282	10.8452	10.5539
0.50	13.5566	13.4398	13.2026	12.7136	12.2027
0.75	15.0440	14.8841	14.5585	13.8816	13.1656
1.00	16.1911	15.9913	15.5829	14.7280	13.8137

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 2.0

RYX	POISONS RATIO				
	0.025	0.05	0.10	0.20	0.30
0.05	7.7096	7.6889	7.6473	7.5632	7.4781
0.25	11.6746	11.6029	11.4580	11.1620	10.8570
0.50	13.9197	13.7972	13.5485	13.0357	12.5000
0.75	15.4129	15.2453	14.9040	14.1948	13.4453
1.00	16.5605	16.3511	15.9236	15.0293	14.0745

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 3.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.8345	7.8131	7.7703	7.6838	7.5962	
0.25	11.7972	11.7237	11.5752	11.2720	10.9598	
0.50	14.0335	13.9034	13.6547	13.1318	12.5861	
0.75	15.5203	15.3496	15.0023	14.2811	13.5199	
1.00	16.6629	16.4502	16.0158	15.1082	14.1403	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 4.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.8973	7.8755	7.8319	7.7438	7.6546	
0.25	11.8491	11.7747	11.6245	11.3179	11.0024	
0.50	14.0793	13.9531	13.6972	13.1699	12.6200	
0.75	15.5625	15.3906	15.0407	14.3145	13.5485	
1.00	16.7027	16.4835	16.0513	15.1382	14.1651	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 5.0

RYX	POISIONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.9314	7.9054	7.8653	7.7763	7.6861	
0.25	11.8751	11.8003	11.6492	11.3408	11.0235	
0.50	14.1018	13.9750	13.7183	13.1885	12.6364	
0.75	15.5830	15.4140	15.0592	14.3306	13.5622	
1.00	16.7218	15.5059	16.0683	15.1526	14.1770	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 6.0

RYX	POISIONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.9516	7.9295	7.8850	7.7954	7.7047	
0.25	11.8898	11.8147	11.8147	11.3537	11.0354	
0.50	14.1143	13.9872	13.7295	13.1988	12.6455	
0.75	15.5944	15.4215	15.0695	14.3395	13.5697	
1.00	16.7324	15.5171	16.0778	15.1603	14.1835	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 7.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.9644	7.9422	7.8975	7.8075	7.7164	
0.25	11.8988	11.8236	11.6717	11.3616	11.0427	
0.50	14.1220	13.9947	13.7366	13.2051	12.6511	
0.75	15.6014	15.4232	15.0758	14.3449	13.5744	
1.00	16.7389	16.5233	16.0835	15.1654	14.1875	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 8.0

RYX	POISONS RATIO					
	0.025	0.05	0.10	0.20	0.30	
0.05	7.9729	7.9506	7.9056	7.8156	7.7242	
0.25	11.9048	11.8295	11.6773	11.3665	11.0475	
0.50	14.1270	13.9996	13.7413	13.2093	12.6547	
0.75	15.6059	15.4326	15.0799	14.3485	13.5774	
1.00	16.7431	16.5274	16.0873	15.1685	14.1901	

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 9.0

* RYX *	POISIONS RATIO					*
* *	0.025	0.05	0.10	0.20	0.30	*
* 0.05 *	7.9789	7.9566	7.9116	7.8212	7.7296	*
* 0.25 *	11.9089	11.8335	11.6812	11.3704	11.0508	*
* 0.50 *	14.1305	14.0030	13.7445	13.2121	12.6572	*
* 0.75 *	15.6090	15.4356	15.0828	14.3509	13.5794	*
* 1.00 *	16.7460	16.5302	16.0899	15.1707	14.1918	*

DESIGN TABLES FOR MODE(1,1) OF FREE PLATE OF ASPECT  
RATIO= 10.0

* RYX *	POISIONS RATIO					*
* *	0.025	0.05	0.10	0.20	0.30	*
* 0.05 *	7.9833	7.9609	7.9160	7.8253	7.7336	*
* 0.25 *	11.9119	11.8354	11.6840	11.3730	11.0532	*
* 0.50 *	14.1330	14.0055	13.7468	13.2142	12.6590	*
* 0.75 *	15.6113	15.4378	15.0848	14.3527	13.5809	*
* 1.00 *	16.7481	16.5322	16.0917	15.1723	14.1931	*

# APPENDIX II

TABLE 6    VALUE OF  $\epsilon_r$

r	Clamped-free	Free-free
1	1.8751041	0
2	4.6940911	0
3	7.8547574	4.7300408
4	10.9955407	7.8532046
5	14.1371684	10.9956078
6	$(2r - 1) \pi/2$	14.1371655
7	for $r > 5$	17.2787594
8		$(2r - 3) \pi/2$
		for $r > 7$

TABLE 7    INTEGRALS OF CHARACTERISTIC FUNCTIONS OF CLAMPED-FREE BEAM

Value of  $1 \int_0^1 \frac{d \phi_r}{dx} \frac{d \phi_s}{dx} dx$

r/s	1	2	3	4	5
1	4.64778	-7.37987	3.94151	-6.59339	4.59198
2		32.41735	-22.35243	13.58245	-22.83952
3			77.29899	-35.64827	20.16205
4				142.90185	-48.71964
5	S Y M M E T R I C A L				228.13325

Value of  $1 \int_0^1 \phi_r \frac{d^2 \phi_s}{dx^2} dx$

r/s	1	2	3	4	5
1	0.85824	-11.74322	27.45315	-37.39025	51.95662
2	1.87385	-13.29425	-9.04222	30.40119	-33.70907
3	1.56451	3.22933	-45.90423	-8.33537	36.38656
4	1.08737	5.54065	4.25360	-98.91821	-7.82895
5	0.91404	3.71642	11.23264	4.73605	-171.58466

TABLE 8 INTEGRALS OF CHARACTERISTIC FUNCTIONS OF FREE-FREE BEAM

Values of  $1 \int_0^1 \frac{d \phi_r}{dx} \frac{d \phi_s}{dx} dx$

r/s	1	2	3	4	5
1	0	0	0	0	0
2	0	12.00000	0	13.85641	0
3	0	0	49.48082	0	35.37751
4	0	13.85641	0	109.92459	0
5	0	0	35.37751	0	186.86671

Values of  $1 \int_0^1 \phi_r \frac{d^2 \phi_s}{dx^2} dx$

r/s	1	2	3	4	5
1	0	0	18.58910	0	43.98096
2	0	0	0	40.59448	0
3	0	0	-12.30262	0	52.58440
4	0	0	0	-46.05012	0
5	0	0	1.80069	0	-98.90480